$\frac{1}{t^{-1}}$ $\frac{1}{t}$ $1 \frac{t}{1}$

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$R = \frac{1}{t^{-1}}$ units $=$ t units

$\therefore R = t$ units

(ii) Find expressions for the coordinates (h, k) of the centre of curvature

$x_1 = h + R\sin\theta$

$h = x_1 - R\sin\theta$

$k = y_1 - R\cos\theta$

$\theta = \tan^{-1}\left[dy/dx\right]$

$\theta = \tan^{-1}\left[\tan t\right]$

$\theta = t$

$x_1 = \cos t + t\sin t$

$h = \cos t + t\sin t - (t)\sin t$

$h = \cos t + t\sin t - t\sin t$

$h = \cos t$

$k = y_1 + R\cos t$

$y_1 = \sin t - t\cos t$

$k = \sin t - t\cos t + (t)\cos t$

$k = \sin t - t\cos t + t\cos t$

$k = \sin t$

Therefore,

Centre of curvature $= (\cos t, \sin t)$

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$x = \cos t + t\sin t$

$y = \sin t - t\cos t$

(i) an expression for the radius of curvature (R)

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$\frac{dx}{dt} = \cancel{\frac{dx}{dt}} - \sin t + t\cos t + \sin t$

$\frac{dx}{dt} = t\cos t$

$\frac{dy}{dt} = \cos t - (t(-\sin t) + \cos t(-1))$

$\frac{dy}{dt} = \cancel{\cos t} + t\sin t - \cancel{\cos t}$

$dy/dt = t\sin t$

$\frac{dy}{dx} = \frac{t\sin t}{t\cos t} = \tan t$

$\frac{d^2y}{dx^2} = \frac{d(\tan t)}{dt} \times \frac{dt}{dx}$

$\frac{d^2y}{dx^2} = \sec^2 t \times \frac{1}{t\cos t}$

$\frac{d^2y}{dx^2} = t^{-1}\sec^3 t$

$R = \frac{\left(1 + (dy/dx)^2\right)^{3/2}}{d^2y/dx^2}$

$= \frac{\left(1 + (\tan t)^2\right)^{3/2}}{t^{-1}\sec^3 t}$

$R = \frac{(1 + \tan^2 t)^{3/2}}{t^{-1}\sec^3 t} = \frac{(\sec t)^{2 \times 3/2}}{t^{-1}\sec^3 t}$

$R = \frac{(\sec t)^3}{t^{-1}(\sec t)^3}$