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1a $\lim_{x \to \frac{\pi}{2}} \left[\frac{(x^2 - \frac{\pi}{4}) \sin(\cos x)}{x - \frac{\pi}{2}} \right] \Rightarrow \lim_{x \to \frac{\pi}{2}} x^2 - \frac{\pi}{4} \times \frac{\sin(\cos x)}{x - \frac{\pi}{2}}$

$= \left(\frac{\pi}{2}\right)^2 - \frac{\pi}{4} \times \frac{\sin(\cos \frac{\pi}{2})}{\frac{\pi}{2} - \frac{\pi}{2}}$

$= \left(\frac{\pi^2}{2^2} - \frac{\pi}{4}\right) \times 0$

$= \left(\frac{\pi^2}{4} - \frac{\pi}{4}\right) \times 0$

$= \frac{\pi^2}{4} - \frac{\pi}{4}$

b $\lim_{x \to \frac{\pi}{2}} \ln \left[\text{Exp} \frac{(3x^2 + 2x - 1)}{x + 1} \right] \Rightarrow \lim_{x \to \frac{\pi}{2}} \ln \left[\text{Exp} \frac{(3x-1)\cancel{(x+1)}}{\cancel{(x+1)}} \right]$

$\Rightarrow \lim_{x \to \frac{\pi}{2}} \ln \left[\text{Exp} \left(3\left(\frac{\pi}{2}\right) - 1\right) \right]$

$\Rightarrow \lim_{x \to \frac{\pi}{2}} \ln \left(\text{Exp} (270 - 1) \right)$

$\Rightarrow \lim_{x \to \frac{\pi}{2}} \ln \left[\text{Exp} (269) \right]$

c $\lim_{x \to 2+\sqrt{3}} \cos \left[\sin^{-1} \frac{x - 2}{x - \sqrt{3}} \right] \Rightarrow \lim_{x \to 2+\sqrt{3}} \cos \left[\sin^{-1} \frac{2 + \sqrt{3} - 2}{2 + \sqrt{3} - \sqrt{3}} \right]$

$\Rightarrow \lim_{x \to 2+\sqrt{3}} \cos \left[\sin^{-1} \frac{\sqrt{3}}{2} \right]$

$\Rightarrow \lim_{x \to 2+\sqrt{3}} \cos (60^\circ)$

$\Rightarrow \frac{1}{2}$

d $\lim_{x \to 4} \left[\frac{x^2 - 8x + 16}{x^2 - 5x + 4} \right] \Rightarrow \lim_{x \to 4} \left[\frac{\cancel{(x-4)}(x-4)}{\cancel{(x-4)}(x-1)} \right]$

$= \lim_{x \to 4} \left[\frac{x-4}{x-1} \right]$

$= \frac{4-4}{4-1} = \frac{0}{3} \Rightarrow \underline{\underline{0}}$

2a $\frac{2}{2 \times 3} + \frac{2}{3 \times 4} + \frac{2}{4 \times 5} + \frac{2}{5 \times 6} + \dots$

$\Rightarrow \frac{2}{2^p} + \frac{2}{3^p} + \frac{2}{4^p} + \frac{2}{5^p} + \dots \Rightarrow \sum_{n=1}^{\infty} \frac{1}{n} P$

$P = 2$

$\therefore$ since; $P > 1$ Therefore, The ~~serie~~ series is converging.

b $\frac{2}{1^2} + \frac{2}{2^2} + \frac{2}{3^2} + \frac{2}{4^2} + \dots$

comparing with $\sum_{n=1}^{\infty} \frac{1}{n} P$

$P = 2$

Since $P > 1$. Therefore, the Series converges.

2c $U_n = \frac{1+2n^2}{1+n^2}$

$\therefore U_{n+1} = \frac{1+2(n+1)^2}{1+(n+1)^2} \Rightarrow \frac{1+2(n^2+2n+1)}{1+(n^2-2n+1)}$

$= \frac{1+2n^2+4n+2}{1+n^2+2n+1}$

$= \frac{2n^2+4n+3}{n^2+2n+2}$.

Applying D'Alembert's Ratio test:

$\frac{U_{n+1}}{U_n} = \frac{2n^2+4n+3}{n^2+2n+2} \times \frac{1+n^2}{1+2n^2}$

$= \frac{2n^2+4n+3+2n^4+4n^3+3n^2}{n^2+2n+2+2n^4+4n^3+4n^2}$

$\lim_{n\to\infty} \frac{U_{n+1}}{U_n} = \frac{2n^4+4n^3+5n^2+4n+3}{2n^4+4n^3+5n^2+2n+2}$

$= \lim_{n\to\infty} \left\{ \frac{2+\frac{4}{n}+\frac{5}{n^2}+\frac{4}{n^3}+\frac{3}{n^4}}{2+\frac{4}{n}+\frac{5}{n^2}+\frac{2}{n^3}+\frac{2}{n^4}} \right\}$

$= \frac{2}{2} = 1$

The series = 1 hence not converging.
Therefore, it is inconclusive.

3 $\frac{x}{27} + \frac{x^2}{125} + \ldots + \frac{x^n}{(2n+1)^3}$

$$|U_n| = \frac{x^n}{(2n+1)^3}$$

$$|U_{n+1}| = \frac{x^{n+1}}{[2(n+1)+1]^3} \Rightarrow \frac{x^{n+1}}{[2n+2+1]^3}$$

$$\therefore |U_{n+1}| = \frac{x^{n+1}}{(2n+3)^3}$$

$$\text{Ratio: } \frac{|U_{n+1}|}{|U_n|} = \frac{x^{n+1}}{(2n+3)^3} \times \frac{(2n+1)^3}{x^n} \qquad x^{n+1} \Rightarrow x^n \cdot x^1$$

$$\frac{|U_{n+1}|}{|U_n|} = \lim_{n \to \infty} \left[\frac{\left(2 + \frac{1}{n}\right)^3 x}{\left(2 + \frac{3}{n}\right)^3}\right]$$

$$= \frac{8x}{8} = x$$

The absolute converging $\lim_{n \to \infty} \left|\frac{U_{n+1}}{U_n}\right| < 1 \quad \therefore |x| < 1$

The range of values of x must be < 1.

4 $\lim_{x \to 0} \left[\frac{\sin x - \cos x}{x^3}\right] \Rightarrow \lim_{x \to 0} \left[\frac{\cos x + \sin x}{3x^2}\right] \Rightarrow$ undefined $\left(\frac{1}{0}\right)$

$$\Rightarrow \lim_{x \to 0} \left[\frac{-\sin x + \cos x}{6x}\right] \Rightarrow \text{undefined} \left(\frac{1}{0}\right)$$

$$\Rightarrow \lim_{x \to 0} \left[\frac{-\cos x + -\sin x}{6}\right]$$

$$\therefore \lim_{x \to 0} \left[\frac{\sin x - \cos x}{x^3}\right] \Rightarrow \frac{-1 - 0}{6} = -\frac{1}{6}$$