ASSIGNMENT ONE

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QUESTION.

$y(0)=5$ and $y'(0)=7$

Solve

$$\frac{d^2y}{dt^2} - \frac{3dy}{dt} + 2y = 2e^{3t}$$

ANSWER

$$\frac{d^2y}{dt^2} - \frac{3dy}{dt} + 2y = 2e^{3t} \quad \text{——— (1)}$$

Recall that;

$$\frac{d^2y}{dt^2} = s^2y(s) - sy(0) - y'(0) \quad \text{———} *$$

$$\frac{dy}{dt} = sy(s) - y(0) \quad \text{———} **$$

$$2e^{3t} = \frac{2}{s-3} \quad \text{———} ***$$

putting equation (*), (**), (***), we have;

$$s^2y(s) - sy(0) - y'(0) - 3(sy(s) - y(0)) + 2y(s) = \frac{2}{s-3}$$

$$s^2y(s) - sy(0) - y'(0) - 3sy(s) + 3y(0) + 2y(s) = \frac{2}{s-3}$$

from the question above, it was given that

$$y(0)=5 \text{ \& } y'(0)=7$$

Input the above into eqn (2)

$$s^2y(s) - s(5) - 7 - 3sy(s) + 3(5) + 2y(s) = \frac{2}{s-3}$$

$$s^2y(s) - 5s - 7 - 3sy(s) + 15 + 2y(s) = \frac{2}{s-3}$$

$$s^2y(s) - 5s - 3sy(s) + 2y(s) + 8 = \frac{2}{s-3}$$

$$s^2y(s) - 3sy(s) + 2y(s) = \frac{2}{s-3} + 5s - 8$$

by factorizing

$$y(s)[s^2 - 3s + 2] = \frac{2}{s-3} + 5s - 8$$

$$y(s)[s^2 - s - 2s + 2] = \frac{2}{s-3} + 5s - 8$$

$$y(s)[(s-1)(s-2)] = \frac{2}{s-3} + 5s - 8$$

divide through by $(s-1)(s-2)$

$$\therefore y(s) = \left[\frac{2}{s-3} + 5s - 8\right] \div [(s-1)(s-2)]$$

$$y(s) = \left[\frac{2 + 5s(s-3) - 8(s-3)}{s-3}\right] \div [(s-1)(s-2)]$$

$$y(s) = \left[\frac{2 + 5s^2 - 15s - 8s + 24}{s-3}\right] \div [(s-1)(s-2)]$$

$$y(s) = \frac{5s^2 - 23s + 26}{s-3} \times \frac{1}{(s-1)(s-2)}$$

$$y(s) = \frac{5s^2 - 23s + 26}{(s-1)(s-2)(s-3)}$$

$$y(s) = \frac{A}{(s-1)} - \frac{B}{(s-2)} + \frac{C}{(s-3)}$$

$$y(s) = \frac{A(s-2)(s-3) + B(s-1)(s-3) + C(s-1)(s-2)}{(s-1)(s-2)(s-3)}$$

$$y(s) = \frac{5s^2 - 23s + 26}{(s-1)(s-2)(s-3)} = \frac{A(s-2)(s-3) + B(s-1)(s-3) + C(s-1)(s-2)}{(s-1)(s-2)(s-3)}$$

$$\therefore 5s^2 - 23s + 26 = A(s-2)(s-3) + B(s-1)(s-3) + C(s-1)(s-2)$$

Let $s = 1$

$$5(1)^2 - 23(1) + 26 = A(1-2)(1-3) + B(1-1)(1-3) + C(1-1)(1-2)$$

$$5(1)^2 - 23(1) + 26 = A(-1)(-2) + B(0)(-2) + C(0)(-1)$$

$$5 - 23 + 26 = 2A$$

$$2A = 8$$

$$A = 8/2 = 4$$

$A = 4$

When $s = 2$

$B(2)^2 - 23(2) + 26 = A(2-2)(2-3) + B(2-1)(2-3) + C(2-1)(2-2)$

$20 - 46 + 26 = A(0)(-1) + B(1)(-1) + C(1)(0)$

$0 = 0 - B + B$

$\therefore B = 0$

When $s = 3$

$5(3)^2 - 23(3) + 26 = A(3-2)(3-3) + B(3-1)(3-3) + C(3-1)(3-2)$

$15 - 69 + 26 = A(1)(0) + B(2)(0) + C(2)(1)$

$2 = 2C$

$C = 2/2$

$\therefore C = 1$

$\therefore A = 4, \ B = 0, \ C = 1$

$$y(s) = \frac{5s^2 - 23s + 26}{(s-1)(s-2)(s-3)} = \frac{4}{(s-1)} - \frac{0}{(s-2)} + \frac{1}{(s-3)}$$

$$y(t) = \mathcal{L}^{-1}\{y(s)\} = \mathcal{L}^{-1}\left\{\frac{4}{(s-1)} - \frac{0}{(s-2)} + \frac{1}{(s-3)}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{4}{s-1}\right\} - \mathcal{L}^{-1}\left\{\frac{0}{(s-2)}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{(s-3)}\right\}$$

$$y(t) = 4e^t + e^{3t}$$