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1. EVALUATE THE FOLLOWING LIMITS OF FUNCTION

(a) $\lim_{x \to \pi/2} \left[\frac{(x^2 - \pi/4) \sin(\cos x)}{x - \pi/2} \right]$

(c) $\lim_{x \to 2+\sqrt{3}} \cos\left(\frac{\sin^{-1}(x-2)}{x - \sqrt{3}} \right)$

b. $\lim_{x \to \pi/2} \ln\left[\exp \frac{(3x^2 + 2x - 1)}{x+1} \right]$

(d) $\lim_{x \to 4} \left(\frac{x^2 - 8x + 16}{x^2 - 5x + 4} \right)$

Soln

(a) $\lim_{x \to \pi/2} \left(\frac{(x^2 - \pi/4) \sin(\cos x)}{x - \pi/2} \right]$

$\lim_{x \to \pi/2} \left(\frac{(\pi/2)^2 - \pi/4 \quad \sin(\cos \pi/2)}{\pi/2 - \pi/2} \right)$

$\left(\frac{2.4674011 - 0.7853981634 \times 0}{0} \right)$

$= \frac{0}{0}$ undefind

Using L'Hopital Rule

$\frac{dy}{dx} \quad -x^2 \cos(\cos x)(\sin x) + \sin(\cos x) 2x$ (Denominator)

$= -\pi/4 \cos(\cos x)(\sin x)$

$\frac{dy}{dx} = 1$ for (numerator)

$$-\frac{\frac{\pi^2}{4} + \pi/4}{1} = 0.1685$$

(b) $\lim_{x \to \pi/2} \ln \left[\exp \frac{(3x^2 + 2x - 1)}{x+1} \right]$

$\ln \left[\exp \frac{(3(\pi/2)^2 + 2(\pi/2) - 1}{\pi/2 + 1} \right]$

$\ln \left[\exp \frac{(3(\frac{3.142}{2})^2 + 2(\frac{3.142}{2}) - 1}{\frac{3.142}{2} + 1} \right]$

$\frac{7.4022 + 2.1416}{\frac{3.142}{2} + 1}$

$= \exp \frac{7.4022 + 2.1416}{2.5708}$

$= \exp \left[\frac{9.5438}{2.5708} \right] = \exp(3.7124)$

$= 10.0913$

c. $\lim_{x \to 2+\sqrt{3}} \cos\left(\sin^{-1} \frac{(x-2)}{(x-\sqrt{3})}\right)$

$\cos\left(\sin^{-1}\left(\frac{(2+\sqrt{3})-2}{(2+\sqrt{3})-\sqrt{3}}\right)\right)$

$\cos\left(\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)$

$\cos(60)$

$= 1/2$ //

d. $\lim_{x \to 4} \left(\frac{x^2 - 8x + 16}{x^2 - 5x + 4} \right)$

$$\frac{(4)^2 - 8(4) + 16}{(4)^2 - 5(4) + 4}$$

$$\frac{16 - 32 + 16}{16 - 20 + 4} = \frac{0}{0} = 0 \quad \text{undefined}$$

Using L'Hopital Rule

$$\frac{dy}{dx} \quad \frac{2x - 8}{2x - 5} = \frac{\cancel{8} - 8}{8 - 5} = \frac{0}{3}$$

$$\frac{d^2y}{dx^2} = \frac{\cancel{2}}{\cancel{2}} = 1$$

2. Determine whether each of the following series is Convergent

a. $\frac{2}{2 \times 3} + \frac{2}{3 \times 4} + \frac{2}{4 \times 5} + \frac{2}{5 \times 6} + \ldots$

nth term $= U_n = \frac{2}{(n+1) \times (2+n)}$ then $U_{(n+1)} = \frac{2}{((n+1)+1) \times (2 + (n+1))}$

$$= \frac{U_{n+1}}{U_n} = \frac{\frac{2}{(n+2)(3+n)}}{\frac{2}{(n+1) \times (2+n)}} = \frac{\cancel{2}}{(n+2)(3+n)} \times \frac{(n+1)(2+n)}{\cancel{2}}$$

$$\lim_{n \to \infty} \frac{U_{n+1}}{U_n} = \frac{(n+1)(2+n)}{(n+2)(3+n)} = \lim_{n \to \infty} \frac{U_{n+1}}{U_n} = \frac{2n + 2n + n^2 + n}{3n + 6 + n^2 + 2n}$$

$$= \frac{2n}{n^2} + \frac{2}{n^2} + \frac{n^2}{n^2} + \frac{n}{n^2} = 2/n + 2/n^2 + 1 + \frac{1}{n}$$

b. $\frac{2}{1^2} + \frac{2}{2^2} + \frac{2}{3^2} + \frac{2}{4^2}$

Using D'Alemberts ratio test

$$U_n = \frac{2}{n^2}$$

$$U_{n+1} = \frac{2}{(n+1)^2}$$

$$\frac{U_{n+1}}{U_n} = \frac{2}{(n+1)^2} \Big/ \frac{2}{n^2}$$

$$\frac{\cancel{2}}{(n+1)^2} \times \frac{n^2}{\cancel{2}} = \frac{n^2}{(n+1)^2}$$

$$\lim_{x \to \infty} \frac{U_{n+1}}{U_n} = \frac{n^2}{(n+1)^2} = \frac{n^2}{n^2+2n+1}$$

$$= \frac{n^2/n^2}{\frac{n^2}{n^2} + \frac{2n}{n^2} + \frac{1}{n^2}}$$

$$= \frac{1}{1 + 2/n + 1/n^2}$$

$$= \frac{1}{1+0+0}$$

$$= 1$$

The series may Converge or deverge Hence making it Inconclusive

ⓒ $U_n = \frac{1+2n^2}{1+n^2}$

Hence $U_{n+1} = \frac{1+2(n+1)^2}{1+(n+1)^2}$

$$U_{n+1} = \frac{1+2(n^2+2n+1)}{1+(n^2+2n+1)}$$

$$U_{n+1} = \frac{1+2n^2+4n+2}{1+n^2+2n+1}$$

$$\frac{U_{n+1}}{U_n} = \frac{\frac{1+2n^2+4n+2}{1+n^2+2n+1}}{\frac{1+2n^2}{1+n^2}}$$

$$\frac{\cancel{1+2n^2}+4n+2}{\cancel{1+n^2}+2n+1} \times \frac{\cancel{1+n^2}}{\cancel{1+2n^2}}$$

$$\frac{U_{n+1}}{U_n} = \frac{4n+2}{2n+1}$$

$$\lim_{n \to \infty} \frac{U_{n+1}}{U_n} = \frac{\frac{4n}{n} + \frac{2}{n}}{\frac{2n}{n} + \frac{1}{n}}$$

$$= \frac{4+0}{2+0}$$

$$= 4/2 = 2$$

The Series diverge because $\lim_{n \to \infty} \frac{U_{n+1}}{U_n} > 1$

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3. Find the range of values of x for which the Series below is absolutely Convergen

$$\frac{x}{27} + \frac{x^2}{125} + \cdots + \frac{x^n}{(2n+1)^3}$$

$\Sigma.(u_n)$ shows absolute Convergent:

$$u_n = \frac{x^n}{(2n+1)^3} \qquad |u_{n+1}| = \frac{x^{n+1}}{(2(n+1)+1)^3} = \frac{x^{n+1}}{(2n+3)^3}$$

$$\frac{u_{n+1}}{u_n} = \frac{\frac{x^{n+1}}{(2n+3)^3}}{\frac{x^n}{(2n+1)^3}}$$

$$= \frac{x^{n+1}}{(2n+3)^3} \times \frac{(2n+1)^3}{x^n}$$

$$= \frac{x^n \cdot x^1}{(2n+3)^3} \times \frac{(2n+1)^3}{x^n} \qquad = \lim_{x \to \infty} \frac{x(2n+1)^3}{(2n+3)^3}$$

$$= \frac{x\left(\frac{2n}{n} + \frac{1}{n}\right)^3}{\left(\frac{2n}{n} + \frac{3}{n}\right)^3}$$

$$x\frac{(2+1/n)^3}{(2+3/n)^3}$$

$$x\frac{(2+0)^3}{(2+0)^3} = \frac{\cancel{8}x}{\cancel{8}}$$

4. Evaluate using L'Hopital Rue

$$\lim_{x\to 0}\left(\frac{\sin x - \cos x}{x^3}\right)$$

$$\frac{dy}{dx}\lim_{x\to 0} = \left(\frac{\cos(0) + \sin(0)}{3(0)}\right) =$$

$$\frac{d^2y}{dx^2}\lim_{x\to 0} = \frac{-\sin x + \cos(0)}{6x}$$

$$\frac{d^3y}{dx^3}\lim_{x\to 0} = \frac{-\cos x + \sin(x)}{6}$$

$$= \frac{-\cos(0) - \sin(0)}{6}$$

$$= \frac{-1-0}{6} = \frac{-1}{6} = -0.1666$$