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The parametric equations of a curve are as given in equations (1) and (2)

$x = \cos t + t\sin t$ —(1)

$y = \sin t - t\cos t$ —(2)

In terms of t determine:

i. An expression for the radius of curvature (R), and

ii Expressions for the coordinates (h, k) of the centre of curvature.

Solution.

Recall,

i $\Rightarrow R = \dfrac{\left[1 + \left(\dfrac{dy}{dx}\right)^2\right]^{3/2}}{\dfrac{d^2y}{dx^2}}$

$x = \cos t + t\sin t$

$y = \sin t - t\cos t$

$\dfrac{dx}{dt} = -\sin t + t\cos t + \sin t$ (1)

$\dfrac{dx}{dt} = t\cos t$

$\dfrac{dy}{dt} = \cos t + t\sin t - \cos t$ (1)

$\dfrac{dy}{dt} = t\sin t$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{dy}{dt} \times \dfrac{dt}{dx} = \dfrac{t\sin t}{t\cos t} = \dfrac{\sin t}{\cos t}$

$\dfrac{d^2y}{dx^2} = \dfrac{d}{dx}\left(\dfrac{dy}{dx}\right) = \dfrac{d}{dt}\left(\dfrac{dy}{dx}\right)\dfrac{dt}{dx}$