Okafor Martins Chinonso.
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Assignment 2.

The Parametric equations of the Curve are given

$$x = \cos t + t \sin t$$
$$y = \sin t - t \cos t$$

i) To find the Radius of Curvature (R) in terms of t.

$$\frac{dx}{dt} = -\sin t + [t \cos t + \sin t]$$
$$= -\sin t + t\cos t + \sin t$$
$$\frac{dx}{dt} = t \cos t$$

$$\frac{dy}{dt} = \cos t - [t \cdot -\sin t + \cos t]$$
$$= \cos t + t\sin t - \cos t$$

$$\frac{dy}{dt} = t \sin t$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{t \sin t}{1} \times \frac{1}{t \cos t}$$

$$\frac{dy}{dx} = \frac{t \sin t}{t \cos t}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dt}\left[\frac{t \sin t}{t \cos t}\right]\frac{dt}{dx}$$

$$\frac{d^2y}{dx^2} = \frac{(t\cos t).(t\cos t + \sin t) + (t\sin t)(-t\sin t + \cos t)}{(t\cos t)^2} \times \frac{dt}{dx}$$

$$= \frac{t^2\cos^2 t + t\cos t\sin t + t^2\sin^2 t - t\cos t\sin t}{(t\cos t)^2} \times \frac{1}{t\cos t}$$

$$= \frac{t^2\cos^2 t + t^2\sin^2 t}{(t\cos t)^2} \times \frac{1}{t\cos t}$$

$$= \frac{t^2(\cos^2 t + \sin^2 t)}{t^2\cos^2 t} \times \frac{1}{t\cos t}$$

Recall $\cos^2 t + \sin^2 t = 1$

$$= \frac{t^2 . 1}{t^2\cos^2 t} \times \frac{1}{t\cos t}$$

$$\frac{d^2y}{dx^2} = \frac{1}{t\cos^3 t}$$

$$R = \frac{\left[1 + \left[\frac{dy}{dx}\right]^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$

$$= \frac{\left[1 + \left[\frac{t\sin t}{t\cos t}\right]^2\right]^{3/2}}{\frac{1}{t\cos^3 t}}$$

$$= \left[\frac{1}{1} + \frac{t^2\sin^2 t}{t^2\cos^2 t}\right]^{3/2} \times \frac{t\cos^3 t}{1}$$

$$= \left[\frac{t^2\cos^2 t + t^2\sin^2 t}{t^2\cos^2 t}\right]^{3/2} \times \frac{t\cos^3 t}{1}$$

$$= \left[\frac{t^2 \cdot 1}{t^2 \cos^2 t}\right]^{3/2} \times t\cos^3 t$$

$$= \left[\frac{1}{\cos^2 t}\right]^{3/2} \times t\cos^3 t$$

$$= \left[\frac{\sqrt{1}}{\sqrt{\cos^2 t}}\right]^{3} \times t\cos^3 t$$

$$= \frac{1^3}{\cos^3 t} \times \frac{t\cos^3 t}{1}$$

$$R = t$$

Therefore, the radius of Curvature in terms of t is

$$R = t$$

ii) For the Coordinates of the Centre of Curvature (h, k)

$$h = x_1 - R\sin\theta$$
$$k = y_1 + R\cos\theta$$

$$\tan\theta = \frac{dy}{dx} = \frac{t\sin t}{t\cos t} = \tan t$$

$$\theta = \tan^{-1}(\tan t$$
$$\theta = t$$

$x_1 = \cos t + t\sin t$ from eqn (i)
$y_1 = \sin t - t\cos t$ from eqn (ii)

$$h = \cos t + t\sin t - [t][\sin t]$$
$$h = \cos t + t\sin t - t\sin t$$
$$h = \cos t$$

$k = \sin t - t\cos t + [t][\cos t]$

$k = \sin t - t\cos t + t\cos t$

$k = \sin t$

Therefore, the expressions in terms of t for the Centre of Curvature (h, k) are

$h = \cos t$ and $k = \sin t$.