1b $\lim_{x \to \pi/2} \ln\left[\exp\left(\frac{3x^2+2x-1}{x+1}\right)\right]$

4

$\frac{3(\pi/2)^2 + 2(\pi/2) - 1}{(\pi/2)+1} = 3.71$

$= \ln\left[\exp(3.71)\right]$

$= \ln\left[40.85\right]$

$= 3.71$

$\lim_{x \to \pi/2} \ln\left[\exp\left(\frac{3x^2+2x-1}{x+1}\right)\right] = 3.71$

1c $\lim_{x \to 2+\sqrt{3}} \cos\left[\sin^{-1}\frac{(x-2)}{(x-\sqrt{3})}\right]$

$\lim_{x \to 2+\sqrt{3}} \cos\left[\sin^{-1}\frac{(2+\sqrt{3}-2)}{(2+\sqrt{3}-\sqrt{3})}\right]$

$\lim_{x \to 2+\sqrt{3}} \cos\left[\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right]$

$\cos(60)$

$\lim_{x \to 2+\sqrt{3}} \cos\left[\sin^{-1}\frac{(x-2)}{(x-\sqrt{3})}\right] = \frac{1}{2}$

1d $\lim_{x \to 4}\left[\frac{x^2-8x+16}{x^2-5x+4}\right]$

$\lim_{x \to 4}\left[\frac{4^2-8(4)+16}{4^2-5(4)+4}\right] = \text{undefined}$

$\lim_{x \to 4}\left[\frac{2x-8}{2x-5}\right]$

$\lim_{x \to 4}\left[\frac{2(4)-8}{2(4)-5}\right] = 0$

16ENG05007 MECHATRONICS

1.

(a) $\lim_{x \to \pi/2} \left[\frac{\left[x^2 - \frac{\pi}{4}\right] \sin(\cos x)}{x - \pi/2} \right]$

$\lim_{x \to \pi/2} \left[\frac{\left((\pi/2)^2 - \pi/4\right) \sin(\cos \pi/2)}{\pi/2 - \pi/2} \right] = \text{undefined}$

$y = \sin(u)$ $\quad u = \cos(x)$

$\frac{dy}{du} = \cos(u)$ $\quad \frac{du}{dx} = -\sin(x)$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \cos(u) \times -\sin(x) = \cos(\cos(x)) \times -\sin(x)$

$f(x) = -x^2 \cos(\cos x) \sin x + 2x \sin(\cos x) + \pi/4 \cos(\cos x) \sin x$

$\lim_{x \to \pi/2} f'(x) = \frac{-\pi^2}{4} \cos\left(\cos\frac{\pi}{2}\right) \sin\left[\frac{\pi}{2}\right] + 2\left[\frac{\pi}{2}\right] \sin\left[\cos\left(\frac{\pi}{2}\right)\right] + \frac{\pi}{4} \cos\left[\cos\left(\frac{\pi}{2}\right)\right] \sin\left[\frac{\pi}{2}\right]$

$= \frac{-\pi^2}{4} \cos(0) \times 1 + \sin(0) \times 180 + \frac{\pi}{4} \cos(1) \times 0$

$= \frac{-\pi^2}{4} \times 1 + \frac{\pi}{4} \times 1$

$= -\frac{\pi^2}{4} + \frac{\pi}{4} = \frac{-\pi^2 + \pi}{4}$

$\lim_{x \to \pi/2} \left[\frac{\left[x^2 - \pi/4\right] \sin(\cos x)}{x - \pi/2} \right] = \frac{-\pi^2 + \pi}{4}$

4

1b $\lim_{x \to \pi/2} \ln\left[\exp\left(\frac{3x^2+2x-1}{x+1}\right)\right]$

$$\frac{3(\pi/2)^2 + 2(\pi/2) - 1}{(\pi/2)+1} = 3.71$$

$$= \ln\left[\exp(3.71)\right]$$

$$= \ln\left[40.85\right]$$

$$= 3.71$$

$$\lim_{x \to \pi/2} \ln\left[\exp\left(\frac{3x^2+2x-1}{x+1}\right)\right] = 3.71$$

1c $\lim_{x \to 2+\sqrt{3}} \cos\left[\sin^{-1}\frac{(x-2)}{(x-\sqrt{3})}\right]$

$$\lim_{x \to 2+\sqrt{3}} \cos\left[\sin^{-1}\frac{(2+\sqrt{3}-2)}{(2+\sqrt{3}-\sqrt{3})}\right]$$

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$$\lim_{x \to 4} \left[\frac{2x-8}{2x-5}\right]$$

$$\lim_{x \to 4} \left[\frac{2(4)-8}{2(4)-5}\right] = 0$$

2a $\frac{2}{2\times3} + \frac{2}{3\times4} + \frac{2}{4\times5} + \frac{2}{5\times6} + \cdots$

taking Standard Series

$\frac{2}{2^p} + \frac{2}{3^p} + \frac{2}{4^p} + \frac{2}{5^p} + \cdots$

when $P = 2$

$\frac{2}{2\times3} < \frac{2}{2^2}$; $\frac{2}{3\times4} < \frac{2}{3^2}$; etc.

it converges.

(b) $\frac{2}{1^2} + \frac{2}{2^2} + \frac{2}{3^2} + \frac{2}{4^2} + \cdots + \frac{2}{n^p}$

Recall if $P > 1$ it converges

if $P \leq 1$ it diverges

$\frac{2}{n^p}$ $P = 2$ $\therefore$ The Series converges

2e $U_n = \frac{1 + 2n^2}{1 + n^2}$

$U_{n+1} = \frac{1 + 2(n+1)^2}{1 + (n+1)^2} = \frac{2n^2 + 4n + 3}{n^2 + 2n + 2}$

$\frac{U_{n+1}}{U_n} = \frac{2n^2+4n+3}{n^2+2n+2} \times \frac{1+n^2}{1+2n^2}$

$= \frac{(1+n^2)(2n^2+4n+3)}{(1+2n^2)(n^2+2n+2)} = \frac{2n^2+4n+3+2n^4+4n^3+3n^2}{n^2+2n+2+2n^4+4n^3+4n^3}$

$= \frac{2n^4+4n^3+5n^2+4n+3}{2n^4+4n^3+5n^2+2n+2}$

$= \frac{4n+3}{2n+3} = \frac{4+3/n}{2+3/n} = \frac{4}{2}$

$\frac{4}{2} > 1$ $\therefore$ it converges.

3. $\frac{x}{27} + \frac{x^2}{125} + \cdots + \frac{x^n}{(2n+1)^3}$

$|U_n| = \frac{x^n}{(2n+1)^3}$

$|U_{n+1}| = \frac{x^{n+1}}{(2n+3)^3}$

$\left|\frac{U_{n+1}}{U_n}\right| = \frac{x^n+1}{(2n+3)^3} \times \frac{(2n+1)^3}{x^n}$

$= \frac{x^{n+1}}{2n^3+18n^2+54n+9} \times \frac{2n^3+6n^2+6n+1}{x^n}$

$= \frac{x(6n^2+6n+1)}{18n^2+54n+9}$

$= \frac{x(6+6/n+1/n^2)}{18+54/n+9/n^2}$

$\left|\frac{U_{n+1}}{U_n}\right| = \frac{6x}{18}$

$\frac{6x}{18} < 1$

$6x < 18$

$|x| < \frac{18}{6}$

$|x| < 3$

4 $$\lim_{x \to 0} \left\{ \frac{\sin x - \cos x}{x^3} \right\}$$

$$\lim_{x \to 0} \left\{ \frac{\sin(0) - \cos(0)}{(0)^3} \right\} = \text{undefined}$$

from L'Hopital Law $\frac{dy}{dx} = \frac{d^2y}{dx}$

$$\lim_{x \to 0} \left\{ \frac{\cos x + \sin x}{3x^2} \right\}$$

$$\lim_{x \to 0} \left\{ \frac{\cos(0) + \sin(0)}{3(0)^2} \right\} = \text{undefined}$$

$$\lim_{x \to 0} \left\{ \frac{-\sin x + \cos x}{6x} \right\}$$

$$\lim_{x \to 0} \left\{ \frac{-\sin(0) + \cos(0)}{6(0)} \right\} = \text{undefined}$$

$$\lim_{x \to 0} \left\{ \frac{-\cos(x) - \sin x}{6} \right\}$$

$$\lim_{x \to 0} \left\{ \frac{-\cos(0) - \sin(0)}{6} \right\} = -\frac{1}{6}.$$