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Matric 16/ENG02/024
DEPT COMPUTER ENGINEERING
COURSE ENG 281

Assignment

(1) Evaluate the following limits of function

a) $\lim_{x \to \frac{\pi}{2}} \left[\frac{(x^2 - \frac{\pi}{4}) \sin(\cos x)}{x - \pi/2} \right]$

Soln

Using L'Hopital's rule

$y = \sin(\cos x)$

Let $y = \sin u$, $u = \cos x$

$\frac{dy}{du} = \cos u \qquad \frac{du}{dx} = -\sin x$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \frac{-\cos u}{\sin x}$

$\frac{dy}{dx} = -\frac{\cos(\cos x)}{\sin x} = -\frac{\cos^2 x}{\sin x}$

Since π is a constant $\pi = 0$

$\frac{dy}{dx} = x^2 - 0/4$

$= \frac{dy}{dx} = 2x$

$\therefore \frac{dy}{dx} = x - 0/2$

$= dy/dx = 1$

$= \frac{dy}{dx} \lim_{x \to \pi/2} \left[(2x) \left[\frac{\frac{-\cos(\cos x)}{\sin x}}{1} \right] \right]$

$= \left[2\left(\frac{\pi}{2}\right) \left(\frac{-\cos(\cos \pi/2)}{\sin \pi/2} \right) \right]$

$= \left[\frac{2\pi}{2} \right] \left[-\cos(\cot \pi/2) \right]$

$= (\pi)(-\cos(\cot \pi/2))$

$= -\pi \cos[\cot \pi/2]$

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b) $\lim_{x \to \pi/2} \ln\left[\exp\frac{(3x^2+2x-1)}{x+1}\right]$

Solu

Factorizing:

$$\ln\left[\frac{\exp(3(\pi/2)-1)(\pi/2+1)}{\pi/2+1}\right]$$

$$= \ln\left[\frac{\exp(3\pi/2-3)(\pi/2+1)}{\pi/2+1}\right]$$

$$= \frac{3\pi}{2} - 3$$

$$= \frac{3\pi-6}{2}$$

c) $\lim_{x \to 2+\sqrt{3}} \cos\left[\sin^{-1}\frac{(x-2)}{(x-\sqrt{3})}\right]$

Solu

$$\cos\left[\sin^{-1}\left(\frac{(2+\sqrt{3})-2}{(2+\sqrt{3})-\sqrt{3}}\right)\right]$$

$$= \cos\left[\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right]$$

$$= \cos(60) = \frac{1}{2}$$

d) $\lim_{x \to 4}\left[\frac{x^2-8x+16}{x^2-5x+4}\right]$

Solu

$$= \frac{(4)^2-8(4)+16}{(4)^2-5(4)+4}$$

$$= \frac{16-32+16}{16-20+4}$$

$$= \frac{0}{0} = 0$$

a) $\frac{2}{2\times 3}$

(2) Determine whether each of the following series is convergent

a) $\frac{2}{2\times3} + \frac{2}{3\times4} + \frac{2}{4\times5} + \frac{2}{5\times6} + \cdots$

nth term $= U_n = \frac{2}{(n+1)\times(2+n)}$, $U_{(n+1)} = \frac{2}{((n+1)+1)\times(2+(n+1))}$

$= \frac{2}{(n+2)(3+n)}$

$\frac{U_{n+1}}{U_n} = \frac{\frac{2}{(n+2)(3+n)}}{\frac{2}{(n+1)(2+n)}}$

$= \frac{2}{(n+2)(3+n)} \times \frac{(n+1)(2+n)}{2}$

$\lim_{n\to\infty} \frac{U_{n+1}}{U_n} = \frac{(n+1)(2+n)}{(n+2)(3+n)}$

$= \lim_{n\to\infty} \frac{U_{n+1}}{U_n} = \frac{2n+2+n^2+n}{3n+6+n^2+2n}$

$= \frac{\frac{2n}{n^2} + \frac{2}{n^2} + \frac{n^2}{n^2} + \frac{n}{n^2}}{\frac{3n}{n^2} + \frac{6}{n^2} + \frac{n^2}{n^2} + \frac{2n}{n^2}}$

$= \frac{2/n + \frac{2}{n^2} + 1 + 1/n}{\frac{3}{n} + \frac{6}{n^2} + 1 + \frac{2}{n}}$

$= \frac{0+0+1+0}{0+0+1+0} = 1$ $\therefore$ The series is Convergent.

(b) $\frac{2}{1^2} + \frac{2}{2^2} + \frac{2}{3^2} + \frac{2}{4^2} + \cdots$

Using D'Alembert's ratio test

$U_n = \frac{2}{n^2}$

$U_{n+1} = \frac{2}{(n+1)^2}$

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$$\frac{U_{n+1}}{U_n} = \frac{\frac{2}{(n+1)^2}}{\frac{2}{n^2}}$$

$$= \frac{2}{(n+1)^2} \times \frac{n^2}{2}$$

$$= \frac{n^2}{(n+1)^2}$$

$$= \lim \frac{U_{n+1}}{U_n} = \frac{n^2}{(n+1)^2} = \frac{n^2}{n^2+2n+1}$$

$$= \frac{\frac{n^2}{n^2}}{\frac{n^2}{n^2} + \frac{2n}{n^2} + 1/n^2} = \frac{1}{1+\frac{2}{n}+\frac{1}{n^2}}$$

$$= \frac{1}{1+0+0} = 1$$ ∴ The series is convergent.

(c) $U_n = \frac{1+2n^2}{1+n^2}$

Solⁿ

$$U_{n+1} = \frac{1+2(n+1)^2}{1+(n+1)^2}$$

$$= U_{n+1} = \frac{1+2(n^2+2n+1)}{1+(n^2+2n+1)}$$

$$= U_{n+1} = \frac{1+2n^2+4n+2}{1+n^2+2n+1}$$

$$= \frac{U_{n+1}}{U_n} = \frac{\frac{1+2n^2+4n+2}{1+n^2+2n+1}}{\frac{1+2n^2}{1+n^2}}$$

$$= \frac{\cancel{1+2n^2}+4n+2}{\cancel{1+n^2}+2n+1} \times \frac{\cancel{1+n^2}}{\cancel{1+2n^2}}$$

$$\frac{U_{n+1}}{U_n} = \frac{4n+2}{2n+1}$$

$$= \lim_{n \to \infty} \frac{U_{n+1}}{U_n} = \frac{4n+2}{2n+1}$$

$$= \frac{U_{n+1}}{U_n} = \frac{4n/n + 2/n}{\frac{2n}{n} + \frac{1}{n}}$$

$$= \frac{4+0}{2+0} = \frac{4}{2} = 2$$ ∴ the series is not convergent $x > 1$

(3) Find the range of values of x for which the series below is absolutely convergent.

$$\frac{x}{27} + \frac{x^2}{125} + \dots + \frac{x^n}{(2n+1)^3}$$

solu

$$|U_n| = \frac{x^n}{(2n+1)^3} \qquad |U_{n+1}| = \frac{x^{n+1}}{(2(n+1)+1)^3} = \frac{x^{n+1}}{(2n+3)^3}$$

$$= \frac{|U_{n+1}|}{|U_n|} = \frac{\frac{x^{n+1}}{(2n+3)^3}}{\frac{x^n}{(2n+1)^3}}$$

$$= \frac{x^{n+1}}{(2n+3)^3} \times \frac{(2n+1)^3}{x^n}$$

$$= \frac{\cancel{x^n} \cdot x^1}{(2n+3)^3} \times \frac{(2n+1)^3}{\cancel{x^n}}$$

$$= \lim_{x \to \infty} \frac{x(2n+1)^3}{(2n+3)^3}$$

$$= \frac{x\left[\frac{2n}{n} + \frac{1}{n}\right]^3}{\left[\frac{2n}{n} + 3/n\right]^3}$$

$$= \frac{x\left[2 + 1/n\right]^3}{\left[2 + 3/n\right]^3}$$

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$$= \frac{x(2+0)^3}{(2+0)^3}$$

$$= \frac{8x}{8} = x$$

Hence $x < 1 \Rightarrow$ The range of values Converge

4) $\lim_{x \to 0} \left[\frac{\sin x - \cos x}{x^3} \right]$

$$= \frac{dy}{dx} \lim_{x \to 0} \left[\frac{\cos x + \sin x}{3x^2} \right]$$

$$= \frac{d^2y}{dx^2} \lim_{x \to 0} \left[\frac{-\sin x + \cos x}{6x} \right]$$

$$\frac{d^3y}{dx^2} \lim_{x \to 0} \left[\frac{-\cos x - \sin x}{6} \right]$$

$$= \left[\frac{-\cos(0) - \sin(0)}{6} \right] = \frac{1}{6}$$

$\therefore$ Since $-1/6 < 1$ it Converges.

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