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16/ENG07/018
CHEMICAL ENGINEERING
ENG281
ENGINEERING MATHEMATICS.

(1) Evaluate the following limits of function:

(a) $\lim_{x \to \frac{\pi}{2}} \left[\frac{\left[x^2 - \frac{\pi}{4}\right] \sin(\cos x)}{x - \frac{\pi}{2}} \right]$

(b) $\lim_{x \to \frac{\pi}{2}} \ln \left(\exp \frac{(3x^2+2x-1)}{x+1} \right)$

(c) $\lim_{x \to 2+\sqrt{3}} \cos \left(\sin^{-1} \frac{(x-2)}{(x-\sqrt{3})} \right)$

(d) $\lim_{x \to 4} \left[\frac{x^2 - 8x + 16}{x^2 - 5x + 4} \right]$

(2) Determine whether each of the following series is convergent.

(a) $\frac{2}{2 \times 3} + \frac{2}{3 \times 4} + \frac{2}{4 \times 5} + \frac{2}{5 \times 6} + \cdots$

(b) $\frac{2}{1^2} + \frac{2}{2^2} + \frac{2}{3^2} + \frac{2}{4^2} + \cdots$

(c) $U_n = \frac{1 + 2n^2}{1 + n^2}$.

(3) Find the range of values of x for which the series below is absolutely convergent.

$$\frac{x}{27} + \frac{x^2}{125} + \cdots + \frac{x^n}{(2n+1)^3}$$

(4) Evaluate using L'Hopital's rule:

$$\lim_{x \to 0} \left(\frac{\sin x - \cos x}{x^3} \right)$$

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Solution

(1) a $\lim_{x \to \frac{\pi}{2}} \left[\frac{(x^2 - \frac{\pi}{4}) \sin(\cos x)}{x - \frac{\pi}{2}} \right]$

Solution for the Numerator.

Let $u = x^2 - \pi/4$.

and $v = \sin(\cos x)$.

$\frac{du}{dx} = 2x$, $\frac{dv}{dx}$ for $v = \sin(\cos x)$.

Let $a = \cos x$.

$\frac{da}{dx} = -\sin x$

$v = \sin a$, $\frac{dv}{da} = \cos a$.

$\frac{dv}{dx} = \frac{dv}{da} \times \frac{da}{dx} = -\sin x \cos a$.

$\frac{dv}{dx} = -\sin x \cos(\cos x) = -\sin x \cos(\cos x)$.

$u\frac{dv}{dx} + v\frac{du}{dx}$.

$= ((x^2 - \pi/4)(-\sin x \cos(\cos x)) + (\sin(\cos x) \cdot 2x)$.

For the denominator.

$y = x - \pi/2$

$\frac{dy}{dx} = 1$.

$\lim_{x \to \pi/2} \left[\frac{(x^2 - \pi/4) \sin \cos(x)}{x - \pi/2} \right] = \lim_{x \to \pi/2} \left[\frac{(x^2 - \pi/4)(-\sin x \cos(\cos x)) + (\sin \cos x) - 2x)}{1} \right]$

$= \left(\frac{\pi}{2}\right)^2 - \frac{\pi}{4} \cdot (\sin \frac{\pi}{2} \cdot \cos(\cos \frac{\pi}{2}) + (\sin(\cos \frac{\pi}{2}) \cdot 2x]$.

$= \left(\frac{\pi}{2}\right)^2 - \frac{\pi}{4} \cdot 1 + 0$

$= \frac{\pi^2}{4} - \frac{\pi}{4}$.

b $\lim_{x \to \frac{\pi}{2}} \ln \left[\operatorname{Exp}\left(\frac{3x^2 + 2x - 1}{x+1} \right) \right]$. $\exp = \frac{1}{\ln}$.

$\lim_{x \to \pi/2} \cancel{\ln} \left[\frac{1}{\ln} \frac{(3x-1)\cancel{(x+1)}}{\cancel{(x+1)}} \right]$

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$\lim_{x \to \pi/2} (3x-1)$

$= 3x-1$

$= 3\left[\frac{\pi}{2}-1\right]$

(c) $\lim_{x \to 2+\sqrt{3}} \cos\left(\frac{\sin^{-1}(x-2)}{(x-\sqrt{3})}\right]$

$\cos\left(\frac{\sin^{-1}(2+\sqrt{3}-2)}{(2+\sqrt{3}-\sqrt{3})}\right)$

$= \cos\left[\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right]$

$= \cos 60$

$= \frac{1}{2}$

d $\lim_{x \to 4} \left[\frac{x^2-8x+16}{x^2-5x+4}\right]$

$= \frac{4^2-8(4)+16}{4^2-5(4)+4}$

$= \frac{16-32+16}{16-20+4} = \frac{0}{0} = 0$

differentiating

$\lim_{x \to 4} \left[\frac{2x-8}{2x-5}\right]$

$= \frac{2(4)-8}{2(4)-5} = \frac{0}{3} = 0$

2 Determine whether each of the following Series is convergent.

(a) $\frac{2}{2\times3} + \frac{2}{3\times4} + \frac{2}{4\times5} + \frac{2}{5\times6} + \cdots$

$= \frac{2}{6} + \frac{2}{12} + \frac{2}{20} + \frac{2}{30} + \cdots$

$r = \frac{\cancel{2}}{1\cancel{2}_2} \times \frac{\cancel{6}}{\cancel{2}} = \frac{1}{2}$

$a = 2/6 = 1/3$

$$S_n = \frac{a(1-r^n)}{1-r}$$

Where $a = 1/3$, and $r = 1/2$ and $n = \infty$

$$S_n = \frac{1/2\,(1 - 1/2^n)}{1 - 1/2}$$

as $n \to \infty$, $\frac{1}{2}^n \to 0$.

$$S_n = \frac{\frac{1}{3}(1-0)}{1 - 1/2}$$

$$S_n = 1/3 \times 2/1$$

$$= 2/3.$$

$\therefore$ The Series Converges Since $2/3$ is a definite Value.

(b) $\frac{2}{1^2} + \frac{2}{2^2} + \frac{2}{3^2} + \frac{2}{4^2} + \cdots$

$\frac{2}{1} + \frac{2}{4} + \frac{2}{9} + \frac{2}{16} + \cdots$

$$r = \frac{2}{4} \div \frac{2}{1}$$

$$= \frac{2}{4} \times \frac{1}{2}.$$

$$r = 1/4$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$a = 2$, $r = 1/4$, $n = \infty$

$$S_n = \frac{2(1 - 1/4^n)}{1 - 1/4}$$

as $n \to \infty$, $1/4 \to 0$.

$$S_n = \frac{2(1-0)}{3/4}$$

$$S_n = \frac{2}{3/4}$$

$$S_n = 8/3$$

$\therefore$ The series converges Since $8/3$ is a definite value.

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c $U_n = \frac{1+2n^2}{1+n^2}$

$\lim_{n\to\infty} U_n = \frac{1+2n^2}{1+n^2}$

$\therefore U_n = \frac{\frac{1}{n^2} + \frac{2n^2}{n^2}}{\frac{1}{n^2} + \frac{n^2}{n^2}} = \frac{\frac{1}{n^2}+2}{\frac{1}{n^2}+1}$

as $n \to \infty$, $\frac{1}{n^2} = 0$.

$U_n = \frac{0+2}{0+1}$, $U_n = 2$.

$\therefore$ The Series is divergent Since $V_n \neq 0$.

(3) $\frac{x}{27} + \frac{x^2}{125} + \cdots + \frac{x^n}{(2n+1)^3}$.

$V_n = \frac{x^n}{(2n+1)^3}$

$U_{n+1} = \frac{x^{n+1}}{(2n+1)^3+1}$

$U_{n+1} = \frac{x^{n+1}}{\{2(n+2)+1)}$

$U_{n+1} = \frac{x^{n+1}}{(2n+3)^3}$

$U_{n+1} = \frac{x^{n+1}}{8n^3+36n^2+54n+27} \div \frac{x^n}{(2n+1)^3}$

$= \frac{x^{n+1}}{8n^3+36n^2+54n+27} \times \frac{(2n+1)^3}{x^n}$

$= \frac{x^n \cdot x^1}{8n^3+36n^2+54n+27} \times \frac{8n^3+12n^2+6n+1}{\cancel{x}^n} = x \times \frac{8n^3+12n^2+6n+1}{8n^3+36n^2+54n+27}$.

$= x\left[\frac{8n^3+12n^2+16n+1}{8n^3+36n^2+54n+27}\right] = x \lim_{n\to\infty}\left[\frac{8+\frac{12}{n}+\frac{6}{n^2}}{8+\frac{36}{n}+\frac{54}{n^2}+\frac{27}{n^3}}\right]$

$= x\left[\frac{8+0+0}{8+0+0+0}\right]$ $\therefore \frac{U_{n+1}}{U_n} = x$, $-1 \leq x < 1$ (it converges).

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(4) $\lim_{x \to 0} \left[\dfrac{\sin x - \cos x}{x^3} \right]$

$= \dfrac{\sin 0 - \cos 0}{0^3}$

$= \dfrac{0-1}{0} = \dfrac{-1}{0}$

differentiating

$= \dfrac{\cos x + \sin x}{3x^2}$

$= \dfrac{\cos 0 + \sin 0}{3(0)^2} = \dfrac{1+0}{0}$ = Undefined.

differentiating again.

$\dfrac{-\sin x + \cos x}{6x} = \dfrac{-\cos x - \sin x}{6}$

$= \dfrac{-\cos 0 - \sin 0}{6} = \dfrac{-1-0}{6} = \dfrac{-1}{6}$

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