ENG 381
Ogbekanmi Gideon. A
Elect Elect
15/ENG04/051

1) $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$

$m^2 - m - 2 = 0$

$m_1 = 2, m_2 = -1$

$y = Ae^{2x} + Be^{-x}$

PI: $y = C$

$\frac{dy}{dx} = 0$

$\frac{d^2y}{dx^2} = 0$

$0 - 0 - 2C = 8$

$-2C = 8$

$C = -8/2$

$C = -4$

PI: $y = -4$

G.S: $y = Ae^{2x} + Be^{-x} - 4$

2) $\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

$m^2 - 4 = 0$

$m^2 = 4$

$m = \pm\sqrt{4}$

$m = \pm 2j$

$y = (\cosh 2x + D\sinh 2x)$

PI: $y = Ce^{3x}$

$\frac{dy}{dx} = 3Ce^{3x}$

$\frac{d^2y}{dx^2} = 9Ce^{3x}$

$9Ce^{3x} - 4(Ce^{3x}) = 10e^{3x}$

$9Ce^{3x} - 4Ce^{3x} = 10e^{3x}$

$5Ce^{3x} = 10e^{3x}$

$C = \frac{10e^{3x}}{5e^{3x}} \therefore C = 2$

P.I: $y = 2e^{3x}$

G.S: $C\cosh 2x + D\sinh 2x + 2e^{3x}$

(3) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}$

$m + 2m + 1 = 0$

$\frac{-2 \pm \sqrt{2^2 - 4\times1\times1}}{2\times1} = \frac{-2}{2} = -1$

$m = -1$

$y = e^{-x}(A + Bx)$

P.I: $y = Ce^{-2x}$

$\frac{dy}{dx} = -2Ce^{-2x}$

$\frac{d^2y}{dx^2} = 4Ce^{-2x}$

$4Ce^{-2x} - 2(2Ce^{-2x}) + Ce^{-2x} = e^{-2x}$

$4Ce^{-2x} - 4Ce^{-2x} + Ce^{-2x} = e^{-2x}$

$Ce^{-2x} = e^{-2x}$

$C = 1$

$y = e^{-2x}$

G.S: $y = e^{-x}(A + Bx) + e^{-2x}$

(4) $\frac{d^2y}{dx^2} + 25y = 5x^2 + x$

$m^2 + 25 = 0$

$m^2 = -25$

$m = \pm\sqrt{-25}$

$m = \pm 5j$

$y = C\cosh 5x + D\sinh 5x$

P.I: $y = Cx^2 + Dx + E$

$\frac{dy}{dx} = 2C(x) + D$

$\frac{d^2y}{dx^2} = 2C$

$2C + 25(Cx^2 + Dx + E) = 5x^2 + x$

$2C + 25Cx^2 + 25Dx + 25E = 5x^2 + x$

$25C = 5$ Comparing coefficients

$\therefore C = 1/5$

6. Cont'd

$y = Ae^x + Be^{-1/3}$

P.I $y = Cx + D$

$\frac{dy}{dx} = C$

$\frac{d^2y}{dx} = 0$

$3(0) - 2C - (Cx + D) = 2x - 3$

$-2C - Cx - D = 2x - 3$

Comparing coefficients

$C = -2$

$-2(-D) = -3$

$-2(-2) - D = -3$

$4 - D = -3$

$D = 4 + 3 = 7$

$y = -2x + 7$

G.S: $y = Ae^x + Be^{-1/3} - 2x + 7$

(8) $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$

$m^2 - 6m + 8 = 0$

$2, 4 = 4x^2 - 2$

$y = Ae^{4x} + Be^{2x} = e^{4x}$

P.I: $y = Cxe^{4x}$

$\frac{dy}{dx} = 4Cxe^{4x} + Ce^{4x}$

$\frac{d^2y}{dx^2} = 16Cxe^{4x} + 8Ce^{4x}$

$16Cxe^{4x} + 8Ce^{4x} - 6(4Cxe^{4x} + Ce^{4x}) + 8(Cxe^{4x})$

$16Cx + 8C - 24Cx - 6C + 8Cx = 8$

$2C = 8$

$C = 4$

$y = 4xe^{4x}$

G.S: $y = Ae^{4x} + Be^{2x} + 4xe^{4x}$

$25D = 1$

$D = 1/25$

$25E + 2C = 0$

$25E + 2(1/5) = 0$

$25E = -2/5$

$E = \frac{-2}{125}$

$y = \frac{1}{5}x^2 + \frac{1}{25}x - \frac{2}{125}$

G.S. $y = (C\cos 5x + D\sin 5x + \frac{1}{5}x^2 + \frac{1}{25}x - \frac{2}{125}$

(5) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$

$m^2 - 2m + 1$

$\frac{2 + \sqrt{(-2)^2 - 4\times 1\times 1}}{2\times 1} = 2/2 = 1$

$m = 1$

$y = e^x(A + Bx)$

$\frac{dy}{dx} = -C\sin x + D\cos x$

$\frac{d^2y}{dx^2} = -C\cos x - D\sin x$

$-C\cos x - D\sin x - 2(-C\sin x + D\cos x) + (C\cos x + D\sin x = 4\sin x$

$-C\cos x - D\sin x + 2C\sin x - 2D\cos x + C\cos x + D\sin x = 4\sin x$

~~$-C\cos x$~~ $- 2D\cos x$ ~~$+ C\cos x$~~ ~~$- D\sin x$~~ $+ 2C\sin x$ ~~$+ D\sin x$~~ $= 4\sin x$

$-2D\cos x + 2C\sin x = 4\sin x$

Comparing coefficients

$-2D = 0$ $\quad 2C = 4$

$D = 0$ $\quad C = 4/2 = 2$

$y = 2\cos x + 0\sin x$

$y = 2\cos x$

G.S. $y = e^x(A + Bx)$

6) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$ given that $x=0$, $y=1$ and $dy/dx = -2$

$m^2 + 4m + 5$

$\frac{-4 \pm \sqrt{4^2 - 4 \times 1 \times 5}}{2 \times 1} = \frac{-4 \pm 2}{2} = -2 \pm j$

$y = e^{-2x}(C\cos 2x + D\sin 2x)$

PI: $y = Ce^{-2x}$

$\frac{dy}{dx} = -2Ce^{-2x}$

$\frac{d^2y}{dx^2} = 4Ce^{-2x}$

$4Ce^{-2x} + 4(-2Ce^{-2x}) + 5(Ce^{-2x}) = 2e^{-2x}$

$4C + 5C - 8C = 2$

$C = 2$, $y = 2e^{-2x}$

GS: $y = e^{-2x}(C\cos 2x + D\sin 2x) + 2e^{-2x}$

When $x = 0$

$y = e^{-2(0)}(C\cos 2(0)) + 2e^{-2(0)}$

$1 = C + 2$

$C = -1$

$\frac{dy}{dx} = e^{-2x}(-2(\sin 2x + 2D\cos 2x) + (C\cos 2x + D\sin 2x) - 4$

at $x = 0$

$-2 = 0 + C$

$-2 = D - 1$

$D = -1$

G.S: $y = e^{-2x}(-C\cos 2x - D\sin 2x)$

7) $3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 2x - 3$

$3m^2 - 2m - 1 = 0$

$\frac{2 \pm \sqrt{(-2)^2 - 4 \times 3 \times 1}}{2 \times 3} = \frac{-2 \pm \sqrt{4+12}}{6}$

$= \frac{2 \pm 4}{6} = \frac{2+4}{6}$ or $\frac{2-4}{6}$

$= 1$ or $-1/3$

$m_1 = 1$, $m_2 = -1/3$