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DEPARTMENT: MECHATRONICS

COURSE: ENG281

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LEEVEL: 200

1

a $$\lim_{x \to \pi/2} \left[\frac{\left[x^2 - \frac{\pi}{4}\right] \sin(\cos x)}{x - \pi/2} \right]$$

$$\lim_{x \to \frac{\pi}{2}} \left[\frac{\left(\left(\pi/2\right)^2 - \pi/4\right) \sin(\cos \pi/2)}{\pi/2 - \pi/2} \right] = \text{undefined}$$

$y = \sin(u)$ $\qquad$ $u = \cos(x)$

$\frac{dy}{du} = \cos(u)$ $\qquad$ $\frac{du}{dx} = -\sin(x)$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \cos(u) \times -\sin(x) = \cos(\cos(x)) \times -\sin(x)$$

$$f'(x) = -x^2 \cos(\cos x) \sin x + 2x \sin(\cos x) + \pi/4 \cos(\cos x) \sin x$$

$$\lim_{x \to \pi/2} f'(x) = -\frac{\pi^2}{4} \cos\left[\cos \frac{\pi}{2}\right] \sin\left[\frac{\pi}{2}\right] + 2\left[\frac{\pi}{2}\right] \sin\left[\cos\left(\frac{\pi}{2}\right)\right] + \frac{\pi}{4} \cos\left[\cos\left[\frac{\pi}{2}\right]\right] \sin\left[\frac{\pi}{2}\right]$$

$$= -\frac{\pi^2}{4} \cos(0) \times 1 + \sin(0) \times 180 + \frac{\pi}{4} \cos(1) \times 0$$

$$= -\frac{\pi^2}{4} \times 1 + \frac{\pi}{4} \times 1$$

$$= -\frac{\pi^2}{4} + \frac{\pi}{4} = \frac{-\pi^2 + \pi}{4}$$

$$\lim_{x \to \pi/2} \left[\frac{\left[x^2 - \pi/4\right] \sin(\cos x)}{x - \pi/2} \right] = \frac{-\pi^2 + \pi}{4}$$

1b $\lim_{x \to \pi/2} \ln\left[\exp\left(\frac{3x^2+2x-1}{x+1}\right)\right]$

$$\# \; \frac{3(\pi/2)^2 + 2(\pi/2) - 1}{(\pi/2)+1} = 3.71$$

$$= \ln\left[\exp(3.71)\right]$$

$$= \ln[40.85]$$

$$= 3.71$$

$$\lim_{x \to \pi/2} \ln\left[\exp\left(\frac{3x^2+2x-1}{x+1}\right)\right] = 3.71$$

1c $\lim_{x \to 2+\sqrt{3}} \cos\left[\sin^{-1}\frac{(x-2)}{(x-\sqrt{3})}\right]$

$$\lim_{x \to 2+\sqrt{3}} \cos\left[\sin^{-1}\left(\frac{2+\sqrt{3}-2}{2+\sqrt{3}-\sqrt{3}}\right)\right]$$

$$\lim_{x \to 2+\sqrt{3}} \cos\left[\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right]$$

$$\cos(60)$$

$$\lim_{x \to 2+\sqrt{3}} \cos\left[\sin^{-1}\frac{(x-2)}{(x-\sqrt{3})}\right] = \frac{1}{2}$$

$x \to 2+\sqrt{3}$ $\left[\qquad (x-\sqrt{3}) \right]$

1d $\lim_{x \to 4} \left[\dfrac{x^2-8x+16}{x^2-5x+4} \right]$

$\lim_{x \to 4} \left[\dfrac{4^2-8(4)+16}{4^2-5(4)+4} \right] = \text{undefined}$

$\lim_{x \to 4} \left[\dfrac{2x-8}{2x-5} \right]$

$\lim_{x \to 4} \left[\dfrac{2(4)-8}{2(4)-5} \right] = 0$

2a $\dfrac{2}{2\times3} + \dfrac{2}{3\times4} + \dfrac{2}{4\times5} + \dfrac{2}{5\times6} + \cdots$

taking standard series

$\dfrac{2}{2^p} + \dfrac{2}{3^p} + \dfrac{2}{4^p} + \dfrac{2}{5^p} + \cdots$

when $p = 2$

$\dfrac{2}{2\times3} < \dfrac{2}{2^2}$; $\dfrac{2}{3\times4} < \dfrac{2}{3^2}$; etc

It Converges

2b $\frac{2}{1^2} + \frac{2}{2^2} + \frac{2}{3^2} + \frac{2}{4^2} + \cdots + \frac{2}{n^P}$

Recall if $P > 1$ it converges

if $P \leq 1$ it diverges

$\frac{2}{n^P}$ $p = 2$ $\therefore$ The series converges

2c $U_n = \frac{1+2n^2}{1+n^2}$

$U_{n+1} = \frac{1+2(n+1)^2}{1+(n+1)^2} = \frac{2n^2+4n+3}{n^2+2n+2}$

$\frac{U_{n+1}}{U_n} = \frac{2n^2+4n+3}{n^2+2n+2} \times \frac{1+n^2}{1+2n^2}$

$= \frac{(1+n^2)(2n^2+4n+3)}{(1+2n^2)(n^2+2n+2)} = \frac{2n^2+4n+3+2n^4+4n^3+3n^2}{n^2+2n+2+2n^4+4n^3+4n^2}$

$= \frac{2n^4+4n^3+5n^2+4n+3}{2n^4+4n^3+5n^2+2n+2}$

$= \frac{4n+3}{2n+3} = \frac{4+3/n}{2+3/n} = \frac{4}{2}$

$\frac{4}{2} > 1$ $\therefore$ It converges

3 $\frac{x}{27} + \frac{x^2}{125} + \cdots + \frac{x^n}{(2n+1)^3} \cdots$

$$|U_n| = \frac{x^n}{(2n+1)^3}$$

$$|U_{n+1}| = \frac{x^{n+1}}{(2n+3)^3}$$

$$\left|\frac{U_{n+1}}{U_n}\right| = \frac{x^{n+1}}{(2n+3)^3} \times \frac{(2n+1)^3}{x^n}$$

$$= \frac{x^{n+1}}{2n^3 + 18n^2 + 54n + 9} \times \frac{2n^3 + 6n^2 + 6n + 1}{x^n}$$

$$= \frac{x(6n^2 + 6n + 1)}{18n^2 + 54n + 9}$$

$$= \frac{x(6 + 6/n + 1/n^2)}{18 + 54/n + 9/n^2}$$

$$\left|\frac{U_{n+1}}{U_n}\right| = \frac{6x}{18}$$

$$\frac{6x}{18} < 1$$

$$6x < 18$$

$$|x| < \frac{18}{6}$$

$$|x| < 3$$

4 $$\lim_{x \to 0} \left\{ \frac{\sin x - \cos x}{x^3} \right\}$$

$$\lim_{x \to 0} \left\{ \frac{\sin(0) - \cos(0)}{(0)^3} \right\} = \text{Undefined}$$

from L' Hopital Law $\frac{dy}{dx} = \frac{d^2y}{dx}$

$$\lim_{x \to 0} \left\{ \frac{\cos x + \sin x}{3x^2} \right\}$$

$$\lim_{x \to 0} \left\{ \frac{\cos(0) + \sin(0)}{3(0)^2} \right\} = \text{undefined}$$

$$\lim_{x \to 0} \left\{ \frac{-\sin(x) + \cos(x)}{6x} \right\}$$

$$\lim_{x \to 0} \left\{ \frac{-\sin(0) + \cos(0)}{6(0)} \right\} = \text{undefined}$$

$$\lim_{x \to 0} \left\{ \frac{-\cos(x) - \sin(x)}{6} \right\}$$

$$\lim_{x \to 0} \left\{ \frac{-\cos(0) - \sin(0)}{6} \right\} = \frac{-1}{6}$$