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Assignment 1

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$$

$$y'' - y' - 2y = 8$$

$$k^2 - k - 2 = 0$$

$$(k^2 - 2k)(+k - 2) = 0$$

$$k(k-2) + 1(k-2) = 0$$

$$(k-2)(k+1) = 0$$

$$k_2 = -1, \quad k_1 = 2$$

$$y = C_1 e^{2x} + C_2 e^{-x}$$

$$y_p = A$$

$$y'_p = 0$$

$$y_p'' = 0$$

$$0 - 0 - 2A = 8$$

$$-2A = 8$$

$$A = \frac{-8}{2} \qquad A = -4 \qquad \therefore y_p = -4$$

$$y_c = y_h + y_p$$

$$y = C_1 e^{2x} + C_2 e^{-x} - 4$$

Question 2

2 $$\frac{d^2y}{dx^2} - 4y = 10e^{3x}$$

$$y'' - 4y = 10e^{3x}$$

$$k^2 - 4 = 0$$

$$k^2 = 4$$

$$k = \pm\sqrt{4}$$

$$k = 2 \text{ or } -2$$

$$k_1 = 2 \text{ and } k_2 = -2$$

$$y_h = C_1 e^{2x} + C_2 e^{-2x}$$

$y_p = Ae^{3x}$

$y'_p = 3Ae^{3x}$

$y''_p = 9Ae^{3x}$

$9Ae^{3x} - 4(Ae^{3x}) = 10e^{3x}$

$9Ae^{3x} - 4Ae^{3x} = 10e^{3x}$

$9A - 4A = 10$

$+5A = 10$

$A = \frac{+10}{5}$

$y_p = \frac{+10}{5} e^{3x} = 2e^{3x}$

$y = y_h + y_p$

$y = c_1 e^{2x} + c_2 e^{-2x} + 2e^{3x}$

Question 3

5 $\frac{\partial^2 y}{\partial x^2} + 2\frac{\partial y}{\partial x} + y = e^{-2x}$

$y'' + 2y' + y = e^{-2x}$

$k^2 + 2k + 1 = 0$

$(k^2 + k)(k + 1) = 0$

$k(k+1) + 1(k+1) = 0$

$(k+1)(k+1) = 0$

$k_1 = -1$ or $k_2 = -1$

$y_h = c_1 y_1 + x c_2 y_2$

$y_h = c_1 e^{-x} + x c_2 e^{-x}$

$y_p = Ae^{-2x}$

$y'_p = -2Ae^{-2x}$

$y''_p = 4Ae^{-2x}$

$4Ae^{-2x} + 2(-2Ae^{-2x}) + Ae^{-2x} = e^{-2x}$

$4Ae^{-2x} - 4Ae^{-2x} + Ae^{-2x} = e^{-2x}$

$Ae^{-2x} = e^{-2x}$

A=1

$y_c = e^{-2x}$

$y = y_h + y_p$

$y = C_1 e^{-x} + C_2 e^{-x} + e^{-2x}$

$y = e^{-x}[C_1 + C_2 x] + e^{-2x}$ //

Question 4

$\frac{d^2y}{dx^2} + 25y = 5x^2 + x$

$y'' + 25y = 5x^2 + x$

$k^2 + 25 = 0$

$k^2 = -25$

$k = \pm\sqrt{-25}$

$k_1 = 5i$ (or) $k_2 = -5i$

$y_h = C_1 e^{5ix} + C_2 e^{-5ix}$

$y_h = A\cos 5x + B\sin 5x$

$y_p = Ax^2 + Bx + C$

$y_p' = 2Ax + B$

$y_p'' = 2A$

$2A + 25Ax^2 + 25Bx + 25C = 5x^2 + x$

$(25A)x^2 + (25B)x + (2A + 25C) = (5)x^2 + (1)x + 0$

$25A = 5$

$A = 5/25 = 1/5$

$25B = 1$

$B = \frac{1}{25}$

$2A + 25C = 0$

$25C = -2A$, $25C = -2\left(\frac{1}{5}\right) = \frac{-2}{5}$

$C = \frac{-2}{5} \times \frac{1}{25} = \frac{-2}{125}$

$y_p = \frac{1}{5}x^2 + \frac{1}{25}x - 2/125$

$y = y_h + y_p$

$y = A\cos 5x + B\sin 5x + \frac{1}{5}x^2 + 1/25x - 2/125$

Question 5

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$$

$$y'' - 2y' + y = 4\sin x$$

$$k^2 - 2k + 1 = 0$$

$$(k^2 - k)(-k + 1) = 0$$

$$k(k-1) - 1(k-1) = 0$$

$$(k-1)(k-1) = 0$$

$$k_1 = 1 \text{ dan } k_2 = 1$$

$$y_h = C_1 y_1 + xC_2 y_2$$

$$y_h = C_1 e^x + xC_2 e^{x6}$$

$$y_p = A\cos x + B\sin x$$

$$y'_p = -A\sin x + B\cos x$$

$$y''_p = -A\cos x - B\sin x$$

$$-A\cos x - B\sin x - 2[-A\sin x + B\cos x] + A\cos x + B\sin x = 4\sin x$$

$$-A\cos x - B\sin x + 2A\sin x - 2B\cos x + A\cos x - B\sin x = 4\sin x$$

$$(2A)\sin x + (-2B)\cos x = (4)\sin x + (0)\cos x$$

$$2A = 4$$

$$A = 4/2 = 2$$

$$-2B = 0$$

$$B = 0$$

$$y_p = 2\cos x + 0\sin x = 2\cos x$$

$$y = y_h + y_p$$

$$y = C_1 e^x + xC_2 e^x + 2\cos x$$

$$y = e^x[C_1 + C_2 x] + 2\cos x$$

Question 6

$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$ given that $x = 0$, $y = 1 \Rightarrow \frac{dy}{dx} = -2$

$y'' + 4y' + 5y = 2e^{-2x}$

$k^2 + 4k + 5 = 0$

$k^2 + 4k = -5$

$k^2 + 4k + (2)^2 = -5 + (2)^2$

$(k+2)^2 = -5 + 4 = -1$

$k + 2 = \pm\sqrt{-1}$

$k = -2 \pm \sqrt{-1}$

$k_1 = -2 + i \Rightarrow k_2 = -2 - i$

$y_h = C_1 e^{(-2+i)x} + C_2 e^{(-2-i)x}$

$y_h = C_1 e^{-2x+ix} + C_2 e^{-2x-ix}$

$y_h = C_1 e^{-2x} e^{ix} + C_2 e^{-2x} e^{-ix}$

$y_h = e^{-2x}[C_1 e^{ix} + C_2 e^{-ix}]$

$y_h = e^{-2x}[A\cos x + B\sin x]$

$y_p = Ae^{-2x}$

$y_p' = -2Ae^{-2x}$

$y_p'' = 4Ae^{-2x}$

$4Ae^{-2x} + 4(-2Ae^{-2x}) + 5(Ae^{-2x}) = 2e^{-2x}$

$4Ae^{-2x} - 8Ae^{-2x} + 5Ae^{-2x} = 2e^{-2x}$

$Ae^{-2x} = 2e^{-2x}$

$A = 2$

$y_p = 2e^{-2x}$

$y = y_h + y_p$

$y = e^{-2x}[A\cos x + B\sin x] + 2e^{-2x}$

$1 = e^{-2(0)}[A\cos(0) + B\sin(0)] + 2e^{-2(0)}$

$1 = A + 2$

$A = 1 - 2 = -1$

$y'' = [-2e^{-2x}[A\cos x + B\sin x]] + [e^{-2x}[-A\sin x + B\cos x]$

$y'' = -2Ae^{-2x}\cos x - 2Be^{-2x}\sin x - Ae^{-2x}\sin x + Be^{-2x}\cos x - 4e^{-2x}$

$-2 = -2Ae^{-2(0)}\cos(0) - 2Be^{-2(0)}\sin(0) - Ae^{-2(0)}\sin(0) + Be^{-2(0)}\cos(0) - 4e^{-2(0)}$

$-2 = -2A + B - 4$

$B = -2 + 2A + 4$

$B = -2 + 2(-1) + 4$

$B = -2 - 2 + 4$

$B = 0$

$-1 = e^{-2x}[-\cos 3x + 0 \sin 3x] + 2e^{-2x}$

$-1 = -e^{-2x}\cos 3x + 2e^{-2x}$

$-1 = e^{-2x}[2 - \cos 3x]$

Question 7

f) $3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 2x - 3$

$3y'' - 2y' - y = 2x - 3$

$3k^2 - 2k - 1 = 0$

$(3k^2 - 3k)(k - 1) = 0$

$3k(k-1) + 1(k-1) = 0$

$(k-1)(3k+1) = 0$

$k_1 = 1$ and $k_2 = \frac{-1}{3}$

$y_h = C_1 e^{x} + C_2 e^{-x/3}$

$y_p = Ax + B$

$y'_p = A$

$y''_p = 0$

$3(0) - 2(A) - (Ax + B) = 2x - 3$

$-2A - Ax - B = 2x - 3$

$(-A)x + (-2A - B) = (2)x + (-3)$

$-A = 2$

$A = -2$

$-2A - B = -3$

$-2(-2) - B = -3$

$B = 4 + 3 = 7$

$y_p = -2x + 7$

$y = y_h + y_p, \quad y = C_1 e^{x} + C_2 e^{-x/3} - 2x + 7$

Question 8

8 $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$

$y'' - 6y' + 8y = 8e^{4x}$

$m^2 - 6m + 8 = 0$

$(m^2 - 4m)(-2m + 8) = 0$

$m(m-4) - 2(m-4) = 0$

$(m-4)(m-2) = 0$

$m_1 = 4 \quad , m_2 = 2$

$y_h = c_1 e^{4x} + c_2 e^{2x}$

$y_p = Axe^{4x}$

$y'_p = A[e^{4x} + 4xe^{4x}] = Ae^{4x} + 4Axe^{4x}$

$y''_p = 4Ae^{4x} + 4A[e^{4x} + 4xe^{4x}]$

$y''_p = 4Ae^{4x} + 4Ae^{4x} + 16xe^{4x} = 8Ae^{4x} + 16Axe^{4x}$

$8Ae^{4x} + 16Axe^{4x} - 6(Ae^{4x} + 4Axe^{4x}) + 8(Axe^{4x}) = 8e^{4x}$

$8Ae^{4x} + 16Axe^{4x} - 6Ae^{4x} - 24Axe^{4x} + 8Axe^{4x} = 8e^{4x}$

$2Ae^{4x} = 8e^{4x}$

$2A = 8$

$A = 4$

$\therefore y_p = 4xe^{4x}$

$y = y_h + y_p$

$y = c_1 e^{4x} + c_2 e^{2x} + 4xe^{4x}$