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DEPARTMENT ELECTRICAL / ELECTRICAL ENGINEERING

MATRIC NO 16 / ENG04 / 001

COURSE ENG 281

QUESTION 1

1. Evaluate the following limits of function:

a. $\lim_{x \to \pi/2} \left[\frac{(x^2 - \pi/4) \sin(\cos x)}{x - \pi/2} \right]$

b. $\lim_{x \to \pi/2} \ln \left[\exp \frac{(3x^2 + 2x - 1)}{x+1} \right]$

c. $\lim_{x \to 2+\sqrt{3}} \cos \left[\sin^{-1} \frac{(x-2)}{(x-\sqrt{3}} \right]$

d. $\lim_{x \to 4} \left[\frac{x^2 - 8x + 16}{x^2 - 5x + 4} \right]$

Solution

$$\lim_{x \to \pi/2} \left[\frac{(x^2 - \pi/4) \sin(\cos x)}{(x - \pi/2)} \right]$$

Using L'Hopital Rule: $f(x)/g(x) = f'(x)/g'(x)$

For Numerator:

$f(x) = (x^2 - \pi/4) \sin(\cos x)$

Applying differentiation:

$dy/dx = u\,dv/dx + v\,du/dx$

$dy/dx = (x^2 - \pi/4) - \sin x \times \cos(\cos x) + \sin(\cos x) \times 2x$

For denominator :

$$x - \pi/2$$

Applying Differentiation: -

$$\frac{dy}{dx} = 1$$

∴ For the entire equation:

$$\lim_{x\to\pi/2}\left[\frac{(x^2-\pi/4)\sin(\cos x)}{(x-\pi/2)}\right] = \lim_{x\to\pi/2}\left[\frac{(x^2-\pi/4)\times -\sin\cos(\cos x) + \sin(\cos x)\times 2x}{1}\right]$$

$\pi/2 = 90^\circ$ (For trigonometric functions)

$$\therefore \lim_{x\to\pi/2}\left[\frac{(x^2-\pi/4)\times -\sin\cos(\cos x) + \sin(\cos x)\times 2x}{1}\right] = \left[\frac{\left((\pi/2)^2-\pi/4\right)\times -\sin\cos(\cos 90) + \sin(\cos 90)\times 2\pi/2}{1}\right]$$

$$\lim_{x\to\pi/2}\left[\frac{(x^2-\pi/4)\sin(\cos x)}{x-\pi/2}\right] = \left[\frac{4807}{1000}\right] = 4.807$$

NO. 1b

$$\lim_{x\to\pi/2} \ln\left[e\,\frac{(3x^2+2x-1)}{x+1}\right]$$

Recall $e = 1/\ln$ (Inverse of $\ln$ or $\log_e$)

Applying to the equation given.

$$\lim_{x\to\pi/2} \ln\left[e\,\frac{(3x^2+2x-1)}{x+1}\right] =$$

$$\lim_{x\to\pi/2} \ln\left(\frac{1}{\ln}\,\frac{(3x^2+2x-1)}{x+1}\right] = \left[\frac{3x^2+2x-1}{x+1}\right] = \left[\frac{(3x-1)(\cancel{x+1})}{\cancel{x+1}}\right] = 3x-1 = 3\pi/2 - 1$$

$$\cancel{\therefore \lim_{x\to\pi/2} \ln\left[e\,\frac{(3x^2+2x-1)}{x+1}\right] = \lim_{x\to\pi/2}\left[\frac{3x^2+2x-1}{x+1}\right] = \left[\frac{3\times(\pi/2)^2 + 2\pi/2 - 1}{\pi/2+1}\right] = \frac{9.543}{2.57}}$$

$$\cancel{\lim_{x\to\pi/2} \ln\left[e\,\frac{(3x^2+2x-1)}{x+1}\right] = \frac{9.543}{2.57} = 3.71}$$

No. 1c

$$\lim_{x \to 2+\sqrt{3}} \cos\left[\sin^{-1}\frac{(x-2)}{(x-\sqrt{3})}\right]$$

solution.

$$\lim_{x \to 2+\sqrt{3}} \cos\left[\sin^{-1}\frac{(x-2)}{(x-\sqrt{3})}\right] = \cos\left[\sin^{-1}\frac{(2+\sqrt{3}-2)}{(2+\sqrt{3}-\sqrt{3})}\right] = \cos(60)$$

$$\lim_{x \to 2+\sqrt{3}} = \cos 60 = 0.5 = \underline{\underline{1/2}}$$

No. 1d

$$\lim_{x \to 4}\left[\frac{x^2-8x+16}{x^2-5x+4}\right] =$$

using L' Hopitals Rule:

$$\lim_{x \to 4}\left[\frac{x^2-8x+16}{x^2-5x+4}\right] = \left[\frac{2x-8}{2x-5}\right] =$$

$$\lim_{x \to 4}\left[\frac{x^2-8x+16}{x^2-5x+4}\right] = \left[\frac{2(4)-8}{2(4)-5}\right] = \underline{\underline{0/3}}$$

No. 2

Determine whether the following Series converges

1. $\frac{2}{2\times3} + \frac{2}{3\times4} + \frac{2}{4\times5} + \frac{2}{5\times6} +$

Sol'n.

$\frac{2}{2\times3} + \frac{2}{3\times4} + \frac{2}{4\times5}$

$\frac{5\cancel{4}3}{67}$

No. 2

Determine whether each of the following series is convergent:

a. $\frac{2}{2\times3} + \frac{2}{3\times4} + \frac{2}{4\times5} + \frac{2}{2\times6} + \cdots$

b. $\frac{2}{1^2} + \frac{2}{2^2} + \frac{2}{3^2} + \frac{2}{4^2} + \cdots$

c. $u_n = \frac{1 + 2n^2}{1 + n^2}$

Sol'n

a. $\frac{2}{2\times3} + \frac{2}{3\times4} + \frac{2}{4\times5} + \cdots$

$$U_n = \frac{x}{(n+1)(n+2)}$$

$$U_{n+1} = \frac{x}{(n+2)(n+3)}$$

$$\lim_{n\to\infty} = \left|\frac{U_{n+1}}{U_n}\right| = \left|\frac{x}{(n+2)(n+3)} \times \frac{(n+2)(n+1)}{x}\right| = \left|\frac{n+1}{n+3}\right|$$

$$\lim_{n\to\infty} = \left|\frac{1+\frac{1}{n}}{1+\frac{3}{n}}\right| = \frac{1}{1} = 1$$

Since $\lim_{n\to\infty}\left|\frac{u_{n+1}}{u_n}\right| = 1$, u_n is not definite and untermined.

No. 2

Determine whether each of the following series is convergent.

2a $\frac{2}{2\times3} + \frac{2}{3\times4} + \frac{2}{4\times5} + \frac{2}{5\times6} + \cdots$

Sol'n

Using D'Alembert's ratio

$$U_n = \frac{2}{n(n+1)}$$

$$U_{n+1} = \frac{2}{(n+1)(n+2)}$$

Ratio $\frac{U_{n+1}}{U_n} = \frac{2}{(n+1)(n+2)} \div \frac{2}{n(n+1)}$

$$= \frac{2}{(n+1)(n+2)} \times \frac{n(n+1)}{2}$$

$$\frac{U_{n+1}}{U_n} = \frac{n}{(n+2)}$$

$$\lim_{n\to\infty} \frac{n/n}{n/n + 2/n} = \frac{1}{1+0} = 1 \qquad p(=1)$$

~~$R \leq 1$~~ $\therefore$ $\lim_{n\to\infty} \frac{U_{n+1}}{U_n} = 1$, is inconclusive

~~$\therefore$ The Series is convergent.~~

2b

$\frac{2}{1^2} + \frac{2}{2^2} + \frac{2}{3^2} + \frac{2}{4^2} + \cdots$

Sol'n

Using D'Alembert's ratio

$$\frac{U_{n+1}}{U_n} = \frac{2}{n^2+2n+1} \div \frac{2}{n^2}$$

$$= \frac{2}{n^2+2n+1} \times \frac{n^2}{2} = \frac{n^2}{n^2+2n+1}$$

$$\lim_{n\to\infty} \frac{n^2/n^2}{n^2/n^2 + 2n/n^2 + 1/n^2} = \frac{1}{1+0+0} = 1$$

~~$= 1$ $(R \leq 1)$~~

~~$R \leq 1$ $\therefore$ The Series is convergent~~

$\lim_{n\to\infty} \frac{U_{n+1}}{U_n} = 1$, the result is inconclusive

$$u_n = \frac{\overset{x}{1 + 2n^2}}{1 + n^2}$$

Divide by the highest power of n

$$\lim_{n \to \infty} \frac{1/n^2 + 2n^2/n^2}{1/n^2 + n^2/n^2} = \frac{0+2}{0+1}$$

$$= 2$$

$u_n \neq 0$ & $P > 1$

$P > 1$ ∴ The series is divergent.

No. 3

Find the range of values of x for which the series below is absolutely convergent.

$$\frac{x}{27} + \frac{x^2}{125} + \cdots + x^n$$

Sol'n

$$\frac{x}{27} + \frac{x^2}{125} + \cdots + \frac{x^n}{(2n+1)^3}$$

$$|u_n| = \frac{x^n}{(2n+1)^3}$$

$$|u_{n+1}| = \frac{x^{n+1}}{(2n+3)^3}$$

$$\left|\frac{u_{n+1}}{u_n}\right| = \frac{x^{n+1}}{(2n+3)^3} \div \frac{x^n}{(2n+1)^3}$$

$$= \frac{x^{n+1}}{(2n+3)^3} \times \frac{(2n+1)^3}{x^n}$$

$$\left|\frac{u_{n+1}}{u_n}\right| = \frac{x(8n^3 + 12n^2 + 6n + 1)}{8n^3 + 36n^2 + 54n + 27}$$

$$= \frac{\cancel{8}x(8n^3/n^3 + 12n^2/n^3 + 6n/n^3 + 1/n^3)}{(8n^3/n^3 + 36n^2/n^3 + 54n/n^3 + 27/n^3)}$$

$$= x\left(8/8\right)$$

$$= x$$ where $(-1 < x < 1)$

Series ~~is convergent when $|x| < 1$~~

For absolute convergency, $\lim_{n \to \infty} \left|\frac{u_{n+1}}{u_n}\right| < 1$ ∴ Series convergent when $\cancel{x} < 1$.

No. 4

Evaluate using L' Hopital's Rule:

$$\lim_{x \to 0} \left[\frac{\sin x - \cos x}{x^3} \right]$$

Sol'n

L' Hopital's Rule: $\dfrac{f(x)}{g(x)} = \dfrac{f'(x)}{g'(x)}$

Let $f(x) = \sin x - \cos x$

$g(x) = x^3$

$f'(x) = \cos x - (-\sin x)$

$= \cos x + \sin x$

$g'(x) = 3x^2$

$$\lim_{x \to 0} \left[\frac{\sin x - \cos x}{x^3} \right] = \left[\frac{\cos x + \sin x}{3x^2} \right]$$

$$= \left[\frac{-\sin x + \cos x}{6x} \right] = \left[\frac{-\cos x - \sin x}{6} \right]$$

$$\lim_{x \to 0} \left[\frac{\sin x - \cos x}{x^3} \right] = \left[\frac{-\cos x - \sin x}{6} \right] = \left[\frac{-\cos(0) - \sin(0)}{6} \right]$$

$$\lim_{x \to 0} \left(\frac{\sin x - \cos x}{x^3} \right) = -\frac{1}{6} //$$