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15/ENG02/031.
COMPT. ENG.

1.) $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$

$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$

$m^2 - m - 2 = 0$

$m^2 + m - 2m - 2 = 0$

$m(m+1) - 2(m+1) = 0$

$m - 2 = 0$

$m + 1 = 0$

$\therefore m_1 = 2$

$m_2 = -1$

$\therefore$ C.F

$y = Ae^{2x} + Be^{-x}$

P.I $= y = C$

$\therefore \frac{dy}{dx} = 0$

$\frac{d^2y}{dx} = 0$

$0 - 0 - 2C = 8$

$-2C = 8$

$C = -4.$

General Solution

$G.S = C.F + B.I$

$y = Ae^{2x} + Be^{-x} - 4$

2.) $\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

$\frac{d^2y}{dx^2} - 4y = 0$

$m^2 - 4 = 0$

$m^2 = 4$

$m = \pm\sqrt{4}$

$m = \pm 2$

$y = A\cosh 2x + B\sinh 2x$

$P.I = y = Ce^{3x}$

$\frac{dy}{dx} = 3Ce^{3x}$

$\frac{d^2y}{dx^2} - 9Ce^{3x}$

$9Ce^{3x} + 0(3Ce^{3x}) - 4(Ce^{3x}) = 10e^{3x}$

$= 9Ce^{3x} - 4Ce^{3x} = 10e^{3x}$

$5Ce^{3x} = 10e^{3x}$

$$C = \frac{10e^{3x}}{5e^{3x}}$$

$$C = 2$$

$$\therefore y = 2e^{3x}$$

$$G.S = C.F + P.I$$

$$y = A\cosh 2x + B\sinh 2x + 2e^{3x}$$

3.) $\frac{d^2y}{dx^2} + \frac{2dy}{dx} + y = e^{-2x}$

$$\frac{d^2y}{dx^2} + \frac{2dy}{dx} + y = 0$$

$$m^2 + 2m + 1 = 0$$

$$m^2 + m + m + 1 = 0$$

$$m(m+1) + 1(m+1) = 0$$

$$m = -1 \text{ (twice)}$$

$\therefore$ C.F $y = e^{-x}(A + Bx)$

$\therefore$ P.I ; $y = Ce^{-2x}$

$$\frac{dy}{dx} = -2Ce^{-2x}$$

$$\frac{d^2y}{dx^2} = 4Ce^{-2x}$$

$$4Ce^{-2x} + 2[-2Ce^{-2x}] + Ce^{-2x} = e^{-2x}$$

$$\cancel{4Ce^{-2x}} - \cancel{4Ce^{-2x}} + Ce^{-2x} = e^{-2x}$$

$c=1$

$\therefore y = 1e^{-2x}$

$y = e^{-2x}$

$\therefore G.S = C.F + P.I$

$y = e^{-x}(A+Bx) + e^{-2x}$

4 $\frac{d^2y}{dx^2} + 25y = 5x^2 + x$

$\frac{d^2y}{dx^2} + 25y = 0$

$m^2 + 25 = 0$

$m^2 + 0 + 25 = 0$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-0 \pm \sqrt{0^2 - 4(1)(25)}}{2(1)}$$

$$\frac{-0 \pm \sqrt{-25 \times 4}}{2}$$

$$\frac{-0 \pm \sqrt{-1} \cdot \sqrt{4} \cdot \sqrt{25}}{2}$$

$$m = \frac{\pm j\, 2\sqrt{25}}{2}$$

$m = \pm j\sqrt{25}$ $\therefore m = \pm j5$

C.F;

$y = A\cos 5x + B\sin 5x$

$P.I = y = Cx^2 + Dx + E$

$\frac{dy}{dx} = 2Cx + D$

$\frac{d^2y}{dx^2} = 2C$

$2c + 0(2cx + 0) + 25(Cx^2 + Dx + E) = 5x^2 + x$

$2c + 25(Cx^2 + Dx + E) = 5x^2 + x$

$2c + 25Cx^2 + 25Dx + 25E = 5x^2 + x$

Comparing Coefficients

$2c + 25E = 0$ --- (i)

$25Cx^2 = 5$

$C = \frac{5}{25}$

$C = 1/5$ --- (ii)

$25D = 1$

$D = \frac{1}{25}$ --- (iii)

Substitute Equ (ii) into (i)

$2\left(\frac{1}{5}\right) + 25E = 0$

$\frac{2}{5} + 25E = 0$

$\frac{1}{25} \times 25E = \frac{-2}{5} \times \frac{1}{25}$

$E = \frac{-2}{125}$

$\therefore P.I = \frac{5}{25}x^2 + \frac{1}{25}x - \frac{2}{125}$

$\frac{-1}{125}\left[25x^2 + 5x - 2\right]$

$\therefore G.S;\ y = A\cos 5x + B\sin 5x + \frac{1}{125}\left[25x^2 + 5x\right.$

5. $\frac{d^2y}{dx^2} - \frac{2dy}{dx} + y = 4\sin x$

$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$

$m^2 - 2m + 1 = 0$

$m^2 - m - m + 1 = 0$

$m(m-1) - m(m-1) = 0$

$(m-1) = 0$ [twice]

$\therefore m = 1$ (twice)

C.F; $y = e^x[A + Bx]$

P.I = $C\cos x + D\sin x = y$

$\frac{dy}{dx} = -C\sin x + D\cos x$

$\frac{d^2y}{dx^2} = -C\cos x - D\sin x$

$[-C\cos x - D\sin x] - 2[-C\sin x + D\cos x] + [C\cos x + D\sin x]$
$= 4\sin x.$

$-C\cos x - D\sin x + 2C\sin x - 2D\cos x + C\cos x + D\sin x$
$= 4\sin x$

$-C\cos x - 2D\cos x + 6\cos x - D\sin x + 2C\sin x + D\sin x = 4\sin x$

$\cos x[-4 - 2D + C] + \sin x[-D + 2C + D] = 4\sin x$

$\cos x[-2D] + \sin x[2C] = 4\sin x$

Comparing Coefficients

$-2D = 0$

$-D = 0$

$2C = 4$

$C = 2$

$\therefore P.T = 2\cos x + 0(\sin x)$

$y = 2\cos x$

$\therefore G.S = y = e^x(A + Bx) + 2\cos x.$

6) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$

Given that a) $x = 0, y = 1$ & $\frac{dy}{dx} = 2$

$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 0.$

$$m^2+4m+5=0$$

$$m=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$m=\frac{-4\pm\sqrt{4^2-4(1)(5)}}{2(1)}$$

$$m=\frac{-4\pm\sqrt{16-20}}{4}$$

$$m=\frac{-4\pm\sqrt{-4}}{2}$$

$$m=\frac{-4}{2}\pm j\frac{2}{2}$$

$$m=-2\pm j$$

$$y=e^{-2x}(A\cos x+B\sin x)$$

$$\therefore \ P I;\ y=Ce^{-2x}$$

$$\frac{dy}{dx}=-2Ce^{-2x}$$

$$\frac{d^2y}{dx^2}=4Ce^{-2x}$$

$$4Ce^{-2x}+4(-2Ce^{-2x})+5(Ce^{-2x})=2e^{-2x}$$

$$4Ce^{-2x}+8Ce^{-2x}+5Ce^{-2x}=2e^{-2x}$$

$$Ce^{-2x}(4-8+5)=2e^{-2x}$$

$$C\cancel{e^{-2x}}=2\cancel{e^{-2x}}$$

$$\therefore \ C=2$$

$$P\cdot I=y=2e^{-2x}$$

$3m+1=0$

$3m=-1$

$m_1=-1/3$

$m-1=0$

$m_2=1$

C.F $y=Ae^{-1/3x}+Be^{x}$

P.I; $y=Cx+D$

$\frac{dy}{dx}=C$

$\frac{d^2y}{dx^2}=0$

$3(0)-2(C)-[(Cx+D)]=2x-3$

$0-2C-Cx-D=2x-3$

$-2C-D-Cx=2x-3$

Comparing Co-efficients

$-2C-D=-3$

$-C=2$

$\therefore C=-2$

$-2(-2)-D=-3$

$4-D=-3$

$-D=3-4$

$-D=-7$

$D=7$

$\therefore$ P.I; $y=-2x+7$

$y = e^{-2x}(A\cos x + B\sin x) + 2e^{-x}$

at $x = 0$, $y = 1$

$\therefore 1 = e^{-2(0)}(A\cos(0) + B\cos(0)) + 2e^{-0}$

$1 = 1(A + 0) + 2$

$1 = A + 2$

$A = -2 + 1$

$A = -1$

$\therefore$ at $x = 0$ $\frac{dy}{dx} = -2$

$\frac{dy}{dx} = -2e^{-2x}(-A\sin x + B\cos(0)) - 2e^{-0}$

$-2 = -2(0 + B) - 2$

$-2 = -2B - 2$

$-2B = -2 + 2$

$B = 0/-2$

$B = 0$

$y = e^{-2x}(-\cos x + 0\sin x) + 2e^{-x}$

$y = -e^{-2x}\cos x + 2e^{-x}$

7.) $3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 2x - 3$

$3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 0$

$3m^2 - 2m - 1 = 0$

-3

$3m^2 - 3m + m - 1 = 0$

$3m(m-1) + 1(m-1) = 0$

G.S

$y = Ae^{-1/3x} + Be^{x} - 2x + 7$

8 $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$

$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 0$

$m^2 - 6m + 8 = 0$

$m^2 - 2m - 4m + 8 = 0$

$m(m-2) - 4(m-2) = 0$

$m - 4 = 0$

$m - 2 = 0$

$m_1 = 4$

$m_2 = 2$

$\therefore y = Ae^{4x} + Be^{2x}$

P.I ; $y = Ce^{4x}$

$\frac{dy}{dx} = 4Ce^{4x}$

$\frac{d^2y}{dx^2} = 16Ce^{4x}$

$16Ce^{4x} - 6(4Ce^{4x}) + 8(Ce^{4x}) = 8e^{4x}$

$Ce^{4x}(16 - 24 + 8) = 8e^{4x}$

$Ce^{4x}(0) = 8e^{4x}$

$C = \frac{8\cancel{e^{4x}}}{\cancel{e^{4x}}(0)}$

$y = 0e^{4x}$

$y = 0$.

G.S; $y = Ae^{4x} + Be^{2x}$