$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$

$m^2 - m - 2 = 0$

$m = 2$ or $m = -1$

$y = Ae^{2x} + Be^{-x}$

$y = c$

$\frac{dy}{dx} = 0 \qquad \frac{d^2y}{dx^2} = 0$

$-2c = 8$

$c = -4$

General solution $= Ae^{2x} + Be^{-x} + (-4)$

$\Rightarrow y = Ae^{2x} + Be^{-x} - 4$

$\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

$m^2 - 4 = 0$

$m^2 = 4$

$m = \pm 2$

$y = Ae^{2x} + Be^{-2x}$

$y = Ce^{3x} \qquad \frac{dy}{dx} = 3Ce^{3x} \qquad \frac{d^2y}{dx^2} = 9Ce^{3x}$

$9Ce^{3x} - 4Ce^{3x} = 10e^{3x}$

$9C - 4C = 10$

$5C = 10$

$C = 2$

$y = 2e^{3x}$

General solution $= y = Ae^{2x} + Be^{-2x} + 2e^{3x}$

3) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}$

$m^2 + 2m + 1 = 0$

$m_1 = -1 \quad m_2 = -1$

$y = e^{-x}\{A + Bx\}$

$y = ce^{-2x} \quad \frac{dy}{dx} = -2ce^{-2x} \quad \frac{d^2y}{dx^2} = 4ce^{-2x}$

$4ce^{-2x} - 4ce^{-2x} + ce^{-2x} = e^{-2x}$

$4c - 4c + c = 1$

$c = 1$

$y = e^{-2x}$

General solution $\Rightarrow y = e^{-x}\{A + Bx\} + e^{-2x}$

4) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = 3x^2 + x$

$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = \tan x$

$m^2 - 2m + 1 = 0$

$m_1 = 1.$

$y = e^{mx}\{A + Bx\}$

$y = C\cos x + D\sin x \quad \frac{dy}{dx} = A + Bx - C\sin x + D\cos x$

$\frac{d^2y}{dx^2} = -C\cos x - D\sin x$

$C\cos x - D\sin x + 2C\sin x - 2D\cos x + C\cos x + D\sin x = \tan x$

$-C\cos x - 2D\cos x + C\cos x = 0$

$-C - 2D + C = 0$

$-2D = 0$

$D = 0$

$-D\sin x + 2C\sin x + D\sin x = 4\sin x$

$-D + 2C + D = 4$

$2C = 4$

$C = 2.$

$y = 2\cos x$

General solution $\Rightarrow y = e^x\{A + Bx\} + 2\cos x.$

7) $\frac{3d^2y}{dx^2} - \frac{2dy}{dx} - 1y = 2x + 3$

$3m^2 - 2m - 1 = 0$

$m_1 = 1 \quad m_2 = -1/3$

$y = Ae^x + Be^{1/3x}$

$y = Cx + D \quad \frac{dy}{dx} = C \quad \frac{d^2y}{dx^2} = 0$

$-2C - Cx - D = 2x - 3$

$-Cx = +2x$

$C = -2$

$-2C - D = -3$

$-2(-2) - D = -3$

$4 - D = -3$

$-D = -7$

$D = 7$

$y = -2x + 7$

General solution:

$y = Ae^x + Be^{1/3x} - 2x + 7.$

9) $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$

$m^2 - 6m + 8 = 0$

$m_1 = 4 \quad m_2 = 2$

$y = Ae^{4x} + Be^{2x}$

$y = Ce^{4x} \quad \therefore y = Cxe^{4x}$

$\frac{dy}{dx} = Cx \cdot 4e^{4x} + e^{4x} \cdot C = 4Cxe^{4x} + Ce^{4x}$

$\frac{d^2y}{dx^2} = Cx \cdot 16e^{4x} + 4e^{4x} \cdot C + e^{4x} \cdot 0 + C \cdot 4e^{4x}$

$= 16Cxe^{4x} + 4Ce^{4x} + 4Ce^{4x}$

$= 16Cxe^{4x} + 8Ce^{4x}$

$16Cxe^{4x} - 8(4Cxe^{4x} + Ce^{4x}) + 8Cxe^{4x} = 8e^{4x}$

$16Cxe^{4x} - 24Cxe^{4x} - 8Ce^{4x} + 8Cxe^{4x} = 8e^{4x}$

$16Cx - 24Cx - 8C + 8Cx = 8$

$16Cx - 24Cx + 8Cx = 0$

$-8C = 8$

$C = -8/8$

$y = -8/8 xe^{4x}$

General solution:

$y = Ae^{4x} + Be^{2x} - 8/8 xe^{4x}$

f) $\frac{d^2y}{dx^2} + 25y = 5x^2 + x$

$m = \pm j5$

$y = A\cos 5x + B\sin 5x$

$y = Cx^2 + Dx + E \quad \frac{dy}{dx} = 2Cx + D \quad \frac{d^2y}{dx^2} = 2C$

$2C + 25Cx^2 + 25Dx + 25E = 5x^2 + x$

$25Cx^2 = 5x^2$

$25Dx = 1x$

$25E + 2C = 0$

$C = \frac{1}{5}$

$D = \frac{1}{25}$

$25E = -2(1/5)$

$E = -\frac{2}{5} \times \frac{1}{25}$

$E = -2/125$

$y = \frac{1}{5}x^2 + \frac{1}{25}x + (-2/125)$

General solution:

$y = A\cos 5x + \sin 5x + \frac{1}{5}x^2 + \frac{1}{25}x - \frac{2}{125}$