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15/ENG03/007
Civil Engineering

ENG 381 (Maths)

Assignment

1) $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$

convert equation into an homogenous equation

$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$

$m^2 - m - 2 = 0$

$(m^2 + m)(-2m - 2) = 0$

$m(m+1) - 2(m+1) = 0$

$(m+1)(m-2) = 0$

$m_1 = -1, m_2 = 2$

$y = Ae^{-x} + Be^{2x}$ (complementary function)

$y = c$

$\frac{dy}{dx} = 0$

$d^2y/dx^2 = 0$

sub into eqn (y, dy/dx & d^2y/dx^2)

$0 - 0 - 2c = 8$

$-2c = 8$

$c = -\frac{8}{2} = -4$

(Particular Integral)

$G.S = C.F + P.I$

$= Ae^{-x} + Be^{2x} - 4$

2.) $\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

$\frac{d^2y}{dx^2} - 4y = 0$

$m^2 - 4 = 0$

$m = \pm\sqrt{4}$

$m = \pm 2$

$y = C\cosh 2x + D\sinh 2x$ (complementary function)

$y = Ce^{3x}$

$\frac{dy}{dx} = 3Ce^{3x}$

$\frac{d^2y}{dx^2} = 9Ce^{3x}$

Sub y, dy/dx & d^2y/dx^2 into eqn

$9Ce^{3x} - 4Ce^{3x} = 10e^{3x}$

$5Ce^{3x} = 10e^{3x}$

$5C = 10$

$C = 2$

P.I, $C = 2e^{3x}$

G.S $= C\cosh 2x + D\sinh 2x + 2e^{3x}$

3.) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}$

$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$

$m^2 + 2m + 1 = 0$

$m^2 + m + m + 1 = 0$

$m(m+1) + 1(m+1) = 0$

$(m+1)$ = twice

$y = e^{-x}(A + Bx)$ (complementary function)

$y = Ce^{-2x}$

$\frac{dy}{dx} = -2Ce^{-2x}$

$d^2y/dx^2 = 4Ce^{-2x}$

sub into eqn

$4Ce^{-2x} + 2(-2Ce^{-2x}) + Ce^{-2x} = e^{-2x}$

$4Ce^{-2x} - 4Ce^{-2x} + Ce^{-2x} = e^{-2x}$

$Ce^{-2x} = e^{-2x}$

$C = 1$

P.I, $C = e^{-2x}$

$G.S = e^{-x}(A+Bx) + e^{-2x}$

4.) $\frac{d^2y}{dx^2} + 25y = 5x^2 + x$

Convert into homogenous equation

$\frac{d^2y}{dx^2} + 25y = 0$

$m^2 + 25 = 0$

$m^2 = -25$

$m = \pm\sqrt{-25}$

$m = \pm j\sqrt{25}$

$m = \pm j5$

$y = C\cos 5x + D\sin 5x$

$y = Cx^2 + Dx + E$

$\frac{dy}{dx} = 2Cx + D$

$d^2y/dx^2 = 2C$

sus into eqn

$2C + 25[Cx^2 + Dx + E] = 5x^2 + x$

$2C + 25Cx^2 + 25Dx + 25E = 5x^2 + x$

$25C = 5$

$C = 1/5$

$25D = 1$

$D = 1/25$

$2C + 25E = 0$

$2(1/5) + 25E = 0$

$25E = -2/5$

$E = -2/125$

$y = 1/5x^2 + 1/25x - 2/125$ (P.I)

$G.S = \cos 5x + D\sin 5x + 1/5x^2 + 1/25x - 2/125$

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5) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 4\sin x$

Convert to homogenous equation

$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$

$m^2 - 2m + 1 = 0$

$m^2 - m - m + 1 = 0$

$m(m-1) - 1(m-1) = 0$

$(m-1) = \text{twice}$

$m = 1$

$y = e^x(A + Bx)$ (P.I)

$y = C\cos x + D\sin x$

$\frac{dy}{dx} = -C\sin x + D\cos x$

$\frac{d^2y}{dx^2} = -C\cos x - D\sin x$

$-C\cos x - D\sin x - 2[-C\sin x + D\cos x] + C\cos x + D\sin x = 4\sin x$

$-C\cos x - D\sin x + 2C\sin x - 2D\cos x + C\cos x + D\sin x = 4\sin x$

$-C\cos x - 2D\cos x + C\cos x - D\sin x + 2C\sin x + D\sin x = 4\sin x$

$\cos x(-C - 2D + C) + \sin x(-D + 2C + D) = 4\sin x$

$-C - 2D + 2C = 0$

$2D = 0$

$D = 0$

$-D + 2C + D = 4$

$2C = 4$

$C = 2$

$y = 2\cos x + D\sin x = 2\cos x$

$G.S = e^x(A + Bx) + 2\cos x$

6) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$ given that $x = 0, y = 1$
and $dy/dx = -2$

Convert ~~to~~ homogenous equation

$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 0$

$m^2 + 4m + 5 = 0$

$a = 1, b = 4, c = 5$

$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$m = \frac{-4 \pm \sqrt{4^2 - 4(1)(5)}}{2 \times 1}$

$m = \frac{-4 \pm \sqrt{16 - 20}}{2}$

$m = \frac{-4 \pm \sqrt{-4}}{2}$

$m = \frac{-4 \pm j\sqrt{4}}{2}$

$m = \frac{-4 \pm j2}{2}$

$m = -2 \pm j$

$y = e^{-2x}(C\cos x + D\sin x)$

$y = Ce^{-2x}$

$dy/dx = -2Ce^{-2x}$

$d^2y/dx^2 = 4Ce^{-2x}$

Sub into eqn

$4Ce^{-2x} + 4(-2C(e^{-2x}) + 5(Ce^{-2x}) = 2e^{-2x}$

$4Ce^{-2x} - 8Ce^{-2x} + 5Ce^{-2x} = 2e^{-2x}$

$4C - 8C + 5C = 2$

$C = 2$

$y = 2e^{-2x}$ (P.I)

~~$y = e$~~

$y = e^{-2x}(C\cos x + D\sin x) + 2e^{-2x}$

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at $x=0$ and $y=1$

$1 = e^{-2(0)}[C\cos 0 + D\sin 0] + 2e^{-2(0)}$

$1 = 1[c+0]+2$

$1 = C+2$

$C = -2+1 = -1$

$\frac{dy}{dx} = [e^{-2x}(-C\sin x + D\cos x)] + [2e^{-2x}(C\cos x + D\sin x)]$
$-4e^{-2x}$

When $\frac{dy}{dx} = -2 \quad x=0$

$-2 = D + [-2C] - 4$

$-2 = D - 2C - 4$

$D - 2C = -2+4$

$D = -2+4+2(-1)$

$D = 0$

$y = e^{-2x}(C\cos x + D\sin x) + 2e^{-2x}$

$= e^{-2x}(-\cos x + 0\sin x) + 2e^{-x}$

$= e^{-2x}(-\cos x + 2)$

$y = e^{-2x}(2-\cos x)$ (Ex.S)

7) $3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 2x-3$

Convert to homogenous equation

$3m^2 - 2m - 1 = 0$

$a=3 \quad b=-2 \quad c=1$

$m = \frac{-(-2) \pm \sqrt{(-2)^2 - 4\times3\times-1}}{2\times3}$

$m = \frac{2 \pm \sqrt{4+12}}{6}$

$m = \frac{2 \pm \sqrt{16}}{6}$

$m = \frac{2 \pm 4}{6}$

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$m = \frac{1 \pm 2}{3}$ $\rightarrow \frac{3}{3} = 1$

$m_1 = 3/3 = 1$ or $m_2 = -1/3$

$y = Ae + Be^{-1/3}$ (C.F)

$y = Cx + D$

$\frac{dy}{dx} = C$

$d^2y/dx^2 = 0$

$3(0) + 2C - (Cx + D) = 2x - 3$

$-2C - Cx - D = 2x - 3$

$-C = 2$

$C = -2$

$-2C - D = -3$

$-2(-2) - D = -3$

$4 - D = -3$

$D = 7$

$y = -2x + 7$ (P.I)

$G.S = Ae + Be^{-1/3} - 2x + 7$

8) $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$

Convert to homogenous equation

$\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 8y = 0$

$m^2 + 6m + 8 = 0$

$m^2 - 2m - 4m + 8 = 0$

$m(m-2) - 4(m-2) = 0$

$(m-2)(m-4) = 0$

$m_1 = 2$, $m_2 = 4$

$y = Ae^{2x} + Be^{4x}$ (C.F)

$y = Cxe^{4x}$

$\frac{dy}{dx} = 4Cxe^{4x} + Ce^{4x}$

$\frac{d^2y}{dx^2} = 16Cxe^{4x} + 4Ce^{4x} + 4Ce^{4x}$

Sub into eqn

$16Cxe^{4x} + 4Ce^{4x} + 4Ce^{4x} - 6(4Cxe^{4x} + Ce^{4x})$
$+ 8(Cxe^{4x}) = 8e^{4x}$

$16Cxe^{4x} + 4Ce^{4x} + 4Ce^{4x} - 24Cxe^{4x} - 6Ce^{4x}$
$+ 8Cxe^{4x} = 8e^{4x}$

$16Cxe^{4x} - 24Cxe^{4x} + 8Cxe^{4x} + 4Ce^{4x} + 4Ce^{4x} -$
$6Ce^{4x} = 8e^{4x}$

$2C\cancel{e^{4x}} = 8\cancel{e^{4x}}$

$2C = 8$

$C = 4$

$P.I, \ y = 4xe^{4x}$

$G.S = Ae^{2x} + Be^{4x} + 4xe^{4x}$

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