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15/ENG06/013
MECHANICAL ENGINEERING

1) $\frac{d^2y}{d\theta^2} + 4\frac{dy}{d\theta} + 5y = 6\sin\theta$

The auxiliary equation becomes

$m^2 + 4m + 5 = 0$

$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$m = \frac{-(4) \pm \sqrt{(4)^2 - 4(1)(5)}}{2(1)} = \frac{-4 \pm \sqrt{16-20}}{2}$

$m = \frac{-4 \pm \sqrt{-4}}{2} = \frac{-4 \pm \sqrt{-1} \times \sqrt{4}}{2} = \frac{-4 \pm 2j}{2}$

$m = -2 \pm j$

$m = \alpha \pm \beta j$

$\alpha = -2, \beta = 1$

The solution to the complementary equation becomes $y = e^{\alpha x}(A\cos\beta\theta + B\sin\beta\theta)$

$y = e^{-2x}(A\cos\theta + B\sin\theta)$

To find the assumed PI.

$(6\sin\theta)\ y = C\cos\theta + D\sin\theta$

$y = C\cos\theta + D\sin\theta$ — ①

$dy/dx = -C\sin\theta + D\cos\theta$ — ②

$d^2y/dx^2 = -C\cos\theta - D\sin\theta$ — ③

Substitute equ ①, ②, ③ into the original equation

$-C\cos\theta - D\sin\theta + 4(-C\sin\theta + D\cos\theta) + 5(\cos\theta + D\sin\theta) = 6\sin\theta$

$-C\cos\theta - D\sin\theta - 4C\sin\theta + 4D\cos\theta + 5C\cos\theta + 5D\sin\theta = 6\sin\theta$

$4C\cos\theta + 4D\sin\theta - 4C\sin\theta + 4D\cos\theta = 6\sin\theta$

$4D\sin\theta + 4C\sin\theta + 4C\cos\theta + 4D\cos\theta = 6\sin\theta$

Comparing equation Coefficients

$4D - 4C = 6$ — ①

$4C + 4D = 0$ — ②

equ ① + ②

$8D = 6$

$D = 6/8 = 3/4$

$\therefore 4D - 4C = 6$

$4(3/4) - 4C = 6$

$3 - 6 = 4C$

$-3 = 4C$

$C = -3/4$

$\therefore$ Assumed $PI = -\frac{3}{4}\cos\theta + \frac{3}{4}\sin\theta$

The general solution to the equation becomes $y = e^{-2x}(A\cos\theta + B\sin\theta) + \frac{3}{4}\sin\theta - \frac{3}{4}\cos\theta$

ii) For the steady state equation

$y = \frac{3}{4}\sin\theta - \frac{3}{4}\cos\theta$

$\frac{dy}{dt} = \frac{3}{4}\cos\theta + \frac{3}{4}\sin\theta$

$\frac{3}{4}\cos\theta + \frac{3}{4}\sin\theta = 0$

$\frac{3}{4}\cos\theta = -\frac{3}{4}\sin\theta$

divide through by $\cos\theta$

$\frac{3}{4}\frac{\cos\theta}{\cos\theta} = -\frac{3}{4}\frac{\sin\theta}{\cos\theta}$

$\frac{3}{4} = -\frac{3}{4}\tan\theta$

$1 = -1\tan\theta$

$\therefore \tan\theta = -1$