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1) $\frac{d^2y}{d\theta^2} + \frac{4dy}{d\theta} + 5y = 6\sin\theta$

The auxiliary equation becomes

$m^2 + 4m + 5 = 0$

$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$m = \frac{(-4) \pm \sqrt{4^2 - 4(1)(5)}}{2(1)}$

$= \frac{-4 \pm \sqrt{16-20}}{2}$

$= \frac{-4 \pm \sqrt{-4}}{2}$

$-4 \pm \sqrt{-1} \times \sqrt{4}$

$\frac{-4 \pm 2j}{\cancel{2}}$

$\cancel{-4}$ $-2 \pm j$

$m = \alpha \pm \beta j$

$\alpha = -2, \beta = 1$

The solution to the complementary equation becomes $y = e^{\alpha x}(A\cos\beta\theta + B\sin\beta\theta)$

$y = e^{-2x}(A\cos\theta + B\sin\theta)$

To find the assumed PI

$(6\sin\theta)$ $y = C\cos\theta + D\sin\theta$

$y = C\cos\theta + D\sin\theta$ —— ①

$dy/dx = -C\sin\theta + D\cos\theta$ —— ②

$d^2y/dx^2 = -C\cos\theta - D\sin\theta$ —— ③

Substitute eqn ①, ②, ③ into the original equation

$-C\cos\theta - D\sin\theta + 4(-C\sin\theta + D\cos\theta) +$

$5(C\cos\theta + D\sin\theta) = 6\sin\theta.$

$-C\cos\theta - D\sin\theta - 4C\sin\theta + 4D\cos\theta + 5$

$C\cos\theta + 5D\sin\theta = 6\sin\theta$

$4C\cos\theta + 4D\sin\theta - 4C\sin\theta + 4D\cos\theta = 6\sin\theta$

$4D\sin\theta - 4C\sin\theta + 4C\cos\theta + 4D\cos\theta = 6\sin\theta$

Comparing coefficients

$4D - 4C = 6$ —— ①

$4C + 4D = 0$ —— ②

eqn ① + eqn ②

$8D = 6$

$D = 6/8 = 3/4$

$4D - 4C = 6$

$4(3/4) - 4C = 6$

$3 - 4C = 6$

$-4C = 6 - 3$

$\frac{-\cancel{4}C}{-\cancel{4}} = \frac{3}{-4}$

∴ Assumed PI = $\frac{-3\cos\theta}{4} + \frac{3}{4}\sin\theta$

The general solution becomes

$y = e^{-2x}(A\cos\theta + B\sin\theta) + \frac{3}{4}\sin\theta - \frac{3}{4}\cos\theta$

ii) For the steady state equation

$y = \frac{3}{4}\sin\theta - \frac{3}{4}\cos\theta$

$dy/d\theta = \frac{3}{4}\cos\theta + \frac{3}{4}\sin\theta$

$\frac{3}{4}\cos\theta + \frac{3}{4}\sin\theta = 0$

$\frac{\frac{3}{4}\cos\theta}{\cos\phi} = -\frac{\frac{3}{4}\sin\theta}{\cos\phi}$

$3/4 = -3/4 \tan\theta$

$1 = -1\tan\theta$

$\tan\theta = -1$