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15 ENAO6 / 067.

Mechanical Engineering.

Assignment 2.

$$\frac{d^2y}{d\theta^2} + 4\frac{dy}{d\theta} + 5y = 6\sin\theta.$$

$$y'' + 4y' + 5y = 6\sin\theta$$

The auxiliary equation becomes:

$$m^2 + 4m + 5 = 0.$$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

$$m = \frac{-4 \pm \sqrt{4^2 - 4(1)(5)}}{2(1)} = \frac{-4 \pm \sqrt{16 - 20}}{2}$$

$$m = \frac{-4 \pm \sqrt{-4}}{2} = \frac{-4 \pm \sqrt{-1} \times \sqrt{4}}{2} = \frac{-4 \pm 2j}{2}$$

$$m = -2 \pm j$$

$$m = \alpha \pm \beta j$$

$$\alpha = -2, \beta = 1.$$

The solution to the complementary equation becomes:

$$y = e^{\alpha x} (A \cos \theta + B \sin \theta).$$

$$y = e^{-2x} (A \cos \theta + B \sin \theta).$$

To find the assumed P.I.

$$(6\sin\theta), y = C \cos \theta + D \sin \theta.$$

$$y = C \cos \theta + D \sin \theta \quad \text{--- (1)}$$

$$dy/dx = -C \sin \theta + D \cos \theta \quad \text{--- (2)}$$

$$d^2y/dx^2 = -C \cos \theta - D \sin \theta \quad \text{--- (3)}$$

Substitute eqn (1), (2), (3) into the Original equation:

$$-C \cos \theta - D \sin \theta + 4(-C \sin \theta + D \cos \theta) + 5(C \cos \theta + D \sin \theta) = 6 \sin \theta.$$

$$-C \cos \theta - D \sin \theta - 4C \sin \theta + 4D \cos \theta + 5C \cos \theta + 5D \sin \theta = 6 \sin \theta.$$

$$4C \cos \theta + 4D \sin \theta - 4C \sin \theta + 4D \cos \theta = 6 \sin \theta.$$

$$4D \sin \theta - 4C \sin \theta + 4C \cos \theta + 4D \cos \theta = 6 \sin \theta.$$

Comparing eqn coefficients.

$$4B - 4C = 6 \quad \text{--- (1)}$$

$$4C + 4B = 0 \quad \text{--- (2)}$$

equ (1) + (2)

$$8B = 6$$

$$B = 6/8 = 3/4$$

$$4B - 4C = 6$$

$$4\left(\frac{3}{4}\right) - 4C = 6$$

$$3 - 6 = 4C$$

$$-3 = 4C$$

$$C = -3/4$$

$\therefore$  Assumed PI =  $-3/4 \cos \theta + 3/4 \sin \theta$

The general solution to the equation becomes

$$y = e^{-2x} (A \cos \theta + B \sin \theta) + 3/4 \sin \theta - 3/4 \cos \theta$$

ii) For steady state equation

$$y = 3/4 \sin \theta - 3/4 \cos \theta$$

$$\frac{dy}{dt} = 3/4 \cos \theta + 3/4 \sin \theta$$

$$3/4 \cos \theta + 3/4 \sin \theta = 0$$

$$3/4 \cos \theta = -3/4 \sin \theta$$

divide thru by  $\cos \theta$

$$3/4 \frac{\cos \theta}{\cos \theta} = \frac{-3}{4} \frac{\sin \theta}{\cos \theta}$$

$$3/4 = -3/4 \tan \theta$$

$$1 = -1 \tan \theta$$

$$\tan \theta = -1$$