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D $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 6\sin\theta$

$y'' + 4y' + 5y = 6\sin\theta$

$K^2 + 4K + 5 = 0$

$K^2 + 4K = -5$

$K^2 + 4K + 2^2 = -5 + 2^2$

$(K+2)^2 = -1$

$(K+2) = \pm\sqrt{-1}$

$K + 2 = \pm i$

$K = \pm i - 2$

$K_1 = +i - 2$

$K_2 = -i - 2$

$Y_1 = Ge^{(-2+i)\theta} + C_2e^{(-2-i)\theta}$

$Y_{11} = Ge^{-2\theta+i\theta} + C_2e^{-2\theta-i\theta}$

$Y_{ii} = e^{-2\theta}Ge^{i\theta} + C_2e^{-2\theta}\cdot e^{i\theta}$

$Y_{ii} = e^{-2\theta}[Ge^{i\theta} + C_2e^{-i\theta}]$

$Y_{ii} = e^{-2\theta}[Ae^{i\theta} + Be^{-i\theta}]$

$Y_{ii} = e^{-2\theta}[A\cos\theta + B\sin\theta]$

$Y_i = A\cos\theta + B\sin\theta$

$y'p = -A\sin\theta + B\cos\theta$

$yp'' = -A\cos\theta - B\sin\theta$

$y'' + 4y' + 5y = 6\sin\theta$

$-A\cos\theta - B\sin\theta + 4(-A\sin\theta + B\cos\theta) + 5(A\cos\theta + B\sin\theta) = 6\sin\theta$

$-A\cos\theta - B\sin\theta - 4A\sin\theta + 4B\cos\theta + 5A\cos\theta + 5B\sin\theta = 6\sin\theta$

$(-A + 4B + 5A)\cos\theta + (-B - 4A + 5B)\sin\theta = 6\sin\theta$

$4A + 4B = 0$

$-4A + 4B = 6$

$8B = 6$

$B = \frac{6}{8} = \frac{3}{4}$

Recall that $4A + 4B = 0$

$4A + 4(\frac{3}{4}) = 0$

$4A + 3 = 0$

$A = -\frac{3}{4}$

$Y_p = -\frac{3}{4}\cos\theta + \frac{3}{4}\sin\theta$

$Y = Y_b + Y_p$

$Y = e^{-2\theta}[A\cos\theta + B\sin\theta] + \frac{3}{4}\sin\theta - \frac{3}{4}\cos\theta$

ii) steady state condition

$Y_p' = 0$

$Y'p = \frac{3}{4}\cos\theta + \frac{3}{4}\sin\theta = 0$

$\frac{3}{4}\cos\theta + \frac{3}{4}\sin\theta = 0$

$\frac{3}{4}\cos\theta = -\frac{3}{4}\sin\theta$

$\frac{\cos\theta}{\cos\theta} = \frac{-\sin\theta}{\cos\theta}$

$1 = -\tan\theta$

$\tan\theta = -1$