Adesopinjo Oladayo Victor
Civil Engineering
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Solutions to Assignment

(1) Evaluate $\lim_{x \to \pi/2} \left[\frac{x^2 - \pi/4 \sin[\cos x}{x - \pi/2}\right]$

$= \frac{[\pi/2]^2 - \pi/4 - \sin[\cos \pi/4]}{\pi/2 - \pi/2} = \frac{0}{0}$ undefined

we differentiate

$= \left[\frac{2x - \sin(-\sin x) + \cos x[\cos x]}{1}\right]$

$= 2[\pi/2] - \sin^2 \pi/2 + \cos^2[\pi/2]$

$= \pi[-1 + 0]$

$= \pi[-1]$

$\therefore \lim_{x \to \pi/2} \left[\frac{x^2 - \pi/2 \sin(\cos x)}{x - \pi/2}\right] = -\pi$

b) $\lim_{x \to \pi/2} \ln\left[\frac{\exp[3x^2 + 2x - 1]}{x+1}\right]$

$\lim_{x \to \pi/2} \left[\frac{(3x-1)(x+1)}{[x+1]}\right]$

$\lim_{x \to \pi/2} = 3x - 1$

$= 3[\pi/2] - 1$

$\frac{3\pi}{2} - 1$

$\lim_{x \to \pi/2} \ln\left[\frac{\exp[3x^2 + 2x - 1]}{x+1}\right] = \frac{3\pi - 2}{2}$

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c) $\lim_{x \Rightarrow 2+\sqrt{3}} \cos\left[\sin^{-1}\frac{[x-2]}{[x-\sqrt{3}]}\right]$

$\cos\left[\sin^{-1}\frac{[2+\sqrt{3}-2]}{[2+\sqrt{3}-\sqrt{3}]}\right]$

$\cos\left[\sin^{-1}\frac{\sqrt{3}}{2}\right]$

$\lim_{x \Rightarrow 2+\sqrt{3}} \cos\left[\sin^{-1}\left[\frac{x-2}{x-\sqrt{3}}\right]\right] = \frac{1}{2}$

d) $\lim_{x \Rightarrow 4}\left[\frac{x^2-8x+16}{x^2-5x+4}\right]$

$= \frac{4^2-[8[4]]+16}{4^2-5[4]+4} = \frac{0}{0}$ undefined

$\lim_{x \Rightarrow 4}\left[\frac{2x-8}{2x-5}\right] \Rightarrow \frac{2[4]-8}{2[4]-5} = \frac{0}{5} = 0$

$\therefore \lim_{x \Rightarrow 4}\left[\frac{x^2-8x+16}{x^2-5x+4}\right] = 0$

2a) $\frac{2}{2\times 3} + \frac{2}{3\times 4} + \frac{2}{4\times 5} + \frac{2}{5\times 6} + \cdots$

$u_n = \frac{2}{(n+1)(n+2)}$ $\quad u_{n+1} = \frac{2}{[n+2][n+3]}$

$\frac{u_{n+1}}{u_n} = \frac{\cancel{2}}{(n+2)(n+3)} \times \frac{(n+1)\cancel{(n+2)}}{\cancel{2}} = \frac{n+1}{n+3}$

$\lim_{n \Rightarrow \infty} \frac{u_{n+1}}{u_n} = \frac{n+1}{n+3}$

divide by highest power of $n$

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$$\lim_{n \to \infty} \frac{U_{n+1}}{U_n} = \frac{1 + 1/n}{1 + 3/n}$$

$$= 1$$

It may be convergent or divergent

From test I $U_n = \lim_{n \to \infty} \cancel{\lim} \frac{2}{(n+1)(n+2)}$

$$U_n = \lim_{n \to 0} = \frac{2}{(n^2+3n+2)}$$

$$U_n = \frac{2/n^2}{\frac{n^2}{n^2} + \frac{3n}{n^2} + \frac{2}{n^2}} = \frac{0}{1} = 0$$

the series is convergent

2b] $\frac{2}{1^2} + \frac{2}{2^2} + \frac{2}{3^2} + \frac{2}{4^2} + \ldots$

$U_n = 2/n^2$ $\quad U_{n+1} = \frac{2}{(n+1)^2}$

$$\lim_{n \to \infty} = \frac{U_{n+1}}{U_n} = \frac{\cancel{2}}{(n+1)^2} \times \frac{n^2}{\cancel{2}} = \frac{n^2}{(n+1)^2} \Rightarrow \frac{n^2}{n^2 + 2n + 1}$$

$$\lim_{n \to \infty} = \frac{n^2/n^2}{n^2/n^2 + 2n/n^2 + 1/n^2} = \frac{1}{1 + 2/n + 1/n^2}$$

$$= \frac{1}{1} = 1$$

From test I

$$\lim_{n \to \infty} U_n = \lim_{n \to \infty} \frac{2}{n^2}$$

$$\lim_{n \to \infty} = \frac{2/n^2}{n^2/n^2} = \frac{0}{1} = 0$$

$\therefore$ The series is convergent.

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2c $U_n = \frac{1+2n^2}{1+n^2}$

$\lim_{n \to \infty} = \frac{1+2n^2}{1+n^2} = \frac{\frac{1}{n^2} + \frac{2n^2}{n^2}}{\frac{1}{n^2} + \frac{n^2}{n^2}}$

$= \frac{0+2}{1} = 2$

$\therefore$ The series is divergent

3) $\frac{x}{27} + \frac{x^2}{125} \cdot \frac{x^n}{(2n+1)^3}$

$U_n = \frac{x^n}{(2n+1)^3}$ $U_{n+1} = \frac{x^{n+1}}{[2n+1+1)^3} = \frac{x^{n+1}}{(2n+2)^3}$

$\lim_{n \to \infty} \frac{U_{n+1}}{U_n} = \frac{x^{n+1}}{(2n+2)^3} \times \frac{(2n+1)^3}{x^n} \Rightarrow \frac{x(2n+1)^3}{(2n+2)^3}$

$\Rightarrow \frac{x[8n^3 + 12n^2 + n + 1]}{8n^3 + 24n^2 + 24n + 8}$

$\cancel{\lim_{n \to \infty}} \Rightarrow \frac{x[\frac{8n^3}{n^3} + \frac{12n^2}{n^3} + \frac{n}{n^3} + \frac{1}{n^3}]}{[\frac{8n^3}{n^3} + \frac{24n^2}{n^3} + \frac{24n}{n^3} + \frac{8}{n^3})}$ $\Rightarrow \frac{x[8 + \frac{12}{n} + \frac{1}{n^2} + \frac{1}{n^3}]}{[8 + \frac{24}{n} + \frac{24}{n^2} + \frac{8}{n^3}]}$

$= \frac{\cancel{x}8}{8} \Rightarrow 8$ $\cancel{x} = 1$ as $n \Rightarrow \infty$

$\frac{1}{n} \Rightarrow 0$

$\cancel{x \leq 1}$ $\frac{8x}{8} \geq x - 1$

$x \leq 1$

4] $\lim_{x \to 0} \left[\frac{\sin x - \cos x}{x^3}\right]$

using L'hopital's Rule

$y = \left[\frac{\sin x - \cos x}{x^3}\right]$

$\frac{dy}{dx} = \left[\frac{\cos x + \sin x}{3x^2}\right]$

$\frac{d^2y}{dx^2} = \frac{-\sin x + \cos x}{6x}$

$\frac{d^3y}{dx^3} = \frac{-\cos x - \sin x}{6}$

$\lim_{x \to 0} = \frac{-\cos 0 - \sin 0}{6} = \frac{-1-0}{6} = -\frac{1}{6}$

$\therefore \lim_{x \to 0} \left[\frac{\sin x - \cos x}{x^3}\right] = -\frac{1}{6}$