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15/ENG07/029
PETROLEUM ENGINEERING
ENG 381

1. $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$

$y'' - y' - 2y = 8 \quad \ldots\ldots \quad (1)$

$y'' - y' - 2y = 0 \quad \ldots\ldots \quad (2)$

$y = e^{kx}, \; y' = ke^{kx}, \; y'' = k^2e^{kx}$ } - subst. into eqn (2)

$k^2e^{kx} - ke^{kx} - 2e^{kx} = 0$

$e^{kx}(k^2 - k - 2) = 0$

$k^2 - k - 2k - 2 = 0$

$k(k+1) - 2(\cancel{k}+1) = 0$

$(k+1)(k-2) \quad \therefore \quad k_1 = -1, \quad k_2 = 2$

$y_1 = e^{k_1x} = e^{-x}, \quad y_2 = e^{k_2x} = e^{2x}$

$y_h = C_1e^{-x} + C_2e^{2x}$

let $y_p = C, \; y_p' = 0, \; y_p'' = 0$ } - subst into eqn (1)

$0 - 0 - 2C = 8$

$-2C = 8$

$C = \frac{8}{-2} = -4$

$y_p = -4$

$\therefore \; y_s = y_h + y_p = C_1e^{-x} + C_2e^{2x} - 4$ //

2. $\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

$y'' - 4y = 10e^{3x} \quad \ldots\ldots \quad (1)$

$y'' - 4y = 0 \quad \ldots\ldots \quad (2)$

let $y = e^{kx}, \; y' = ke^{kx}, \; y'' = k^2e^{kx}$ } - subst into eqn (2)

$k^2e^{kx} - 4e^{kx} = 0$

$e^{kx}(k^2 - 4) = 0$

$k^2 - 4 = 0, \quad k^2 = 4 \quad \therefore \quad k = \pm\sqrt{4}, \quad k = \pm 2$

$y_h = C_1\cosh 2x + C_2\sinh 2x$

let $y_p = Ae^{3x} \quad y_p' = 3Ae^{3x} \quad y_p'' = 9Ae^{3x}$ } - subs into eqn (1)

$9A\cancel{e^{3x}} - 4A\cancel{e^{3x}} = 10\cancel{e^{3x}}$

$5A = 10$

$A = 2$

$y_p = Ae^{3x} = 2e^{3x}$

$y_s = y_h + y_p = C_1 \cosh 2x + C_2 \sinh 2x + 2e^{3x}$ //

3 $\frac{d^2y}{dx^2} + \frac{2dy}{dx} + y = e^{-2x}$

$y'' + 2y' + y = e^{-2x}$ .... ①

$y'' + 2y' + y = 0$ ..... ②

let $y = e^{kx}$ $y' = ke^{kx}$ $y'' = k^2e^{kx}$ } - Subst into eqn ②

$k^2e^{kx} + 2ke^{kx} + e^{kx} = 0$

$e^{kx}(k^2 + 2k + 1) = 0$

$k^2 + k + k + 1 = 0$

$k(k+1) + 1(k+1) = 0$

$k = -1$ twice

$y = e^{kx} = e^{-x}$

$y_h = (C_1 + xC_2)e^{-x}$

let $y_p = Ae^{-2x}$ ~~y_p'~~ $y_p' = -2Ae^{-2x}$ $y_p'' = 4Ae^{-2x}$ } - Subst into eqn ①

$4Ae^{-2x} + 2(-2Ae^{-2x}) + Ae^{-2x} = e^{-2x}$

$4A\cancel{e^{-2x}} - 4A\cancel{e^{-2x}} + A\cancel{e^{-2x}} = \cancel{e^{-2x}}$

$4A - 4A + A = 1$

$\therefore A = 1$

$y_p = Ae^{-2x} = e^{-2x}$

$y_s = y_h + y_p = e^{-x}(C_1 + xC_2) + e^{-2x}$ //

4 $\frac{d^2y}{dx^2} + 25y = 5x^2 + x$

$y'' + 25y = 5x^2 + x$ ..... ①

$y'' + 25y = 0$ ... ②

let $y = e^{kx}$ $y' = ke^{kx}$ $y'' = k^2e^{kx}$ } - Subst into eqn ②

$k^2e^{kx} + 25e^{kx} = 0$

$e^{kx}(k^2 + 25) = 0$

$k^2 + 25 = 0$; $k^2 = -25$

$k = \sqrt{+25} \times \sqrt{-1}$

$k = \pm 5j$ $k_1 = 5i$ $k_2 = -5i$

$y_1 = e^{k_1 x} = e^{5ix}$ $\quad y_2 = e^{k_2 x} = e^{-5ix}$

$y_h = A\cos 5x + B\sin 5x$

let $y_p = Ax^2 + Bx + c$, $y_p' = 2Ax + B$ $\quad y_p'' = 2A$ — Subst into eqn ①

$2A + 25(Ax^2 + Bx + c) = 5x^2 + x$

$2A + 25Ax^2 + 25Bx + 25C = 5x^2 + x$

$25A = 5 \quad \therefore A = \frac{1}{5}$

$25B = 1 \quad \therefore B = \frac{1}{25}$

$2A + 25C = 0 \quad \therefore C = -\frac{2}{125}$; $y_p = \frac{1}{5}x^2 + \frac{1}{25}x - \frac{2}{125}$

$y_s = y_h + y_p$

$y_s = A\cos 5x + B\sin 5x + \frac{1}{5}x^2 + \frac{1}{25} - \frac{2}{125}$

5 $\quad \frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$

$y'' - 2y' + y = 4\sin x$ .... ①

$y'' - 2y' + y = 0$ ...... ②

let $y = e^{kx}$ $\quad y' = ke^{kx}$ $\quad y'' = k^2 e^{kx}$ $\Rightarrow$ Subst into eqn ②

$k^2 e^{kx} - 2ke^{kx} + e^{kx} = 0$

$e^{kx}(k^2 - 2k + 1) = 0$

$k^2 - k - k + 1 = 0$

$k(k-1) - 1(k-1)$

$\therefore k = 1$ twice

$y = e^{kx} = e^{x}$

$y_h = (C_1 + xC_2)e^{x}$

let $y_p = A\sin x + B\cos x$ $\quad y_p' = A\cos - B\sin x$ $\quad y_p'' = -A\sin x - B\cos x$

$-A\sin x - B\cos x - 2(A\cos x - B\sin x) + A\sin x + B\cos x = 4\sin x$

$-A\sin x - B\cos x - 2A\cos x + 2B\sin x + A\sin x + B\cos x = 4\sin x$

$(-A + 2B + A)\sin x + (-B - 2A + B)\cos x = 4\sin x + 0\cos x$

$-A + 2B + A = 4$

$-B - 2A + B = 0$

$B = 2$, $A = 0$

$y_p = A\sin x + B\cos x$

$y_p = 2\cos x$

$y_s = y_h + y_p = (C_1 + xC_2)e^{x} + 2\cos x$ //

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6 $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$

$y'' + 4y' + 5y = 2e^{-2x}$ .... (1)

$y'' + 4y' + 5y = 0$ .... (2)

$y = e^{kx}$ $\quad y' = ke^{kx}$ $\quad y'' = k^2e^{kx}$ } - Subst into eqn (2)

$k^2e^{kx} + 4ke^{kx} + 5e^{kx} = 0$

$e^{kx}(k^2 + 4k + 5) = 0$

$k_1 = -2 + i$

$k_2 = -2 - i$

$y_1 = e^{k_1x} = e^{(-2+i)x} = e^{-2x} \cdot e^{ix}$

$y_2 = e^{k_2x} = e^{(-2-i)x} = e^{-2x} \cdot e^{-ix}$

$y = c_1y_1 + c_2y_2 = c_1e^{-2x}e^{ix} + c_2e^{-2x} \cdot e^{-ix}$

$= e^{-2x}(c_1e^{ix} + c_2e^{-ix})$

$y_h = e^{-2x}(A\cos x + B\sin x)$

let $y_p = ce^{-2x}$ $\quad y_p' = -2ce^{-2x}$ $\quad y_p'' = 4ce^{-2x}$

$4ce^{-2x} + 4(-2ce^{-2x}) + 5(ce^{-2x}) = 2e^{-2x}$

$4ce^{-2x} - 8ce^{-2x} + 5ce^{-2x} = 2e^{-2x}$

$4c - 8c + 5c = 2$

$c = 2$

$y_p = ce^{-2x} = 2e^{-2x}$

$y_s = y_h + y_p$

$y_s = e^{-2x}(A\cos x + B\sin x) + 2e^{-2x}$

$y_s' = A(-2e^{-2x}\cos x - e^{-2x}\sin x) + B(-2e^{-2x}\sin x + e^{-2x}\cos x] - 4e^{-2x}$

at $x = 0$, $y = 1$, $y' = -2$

$1 = e^{-2(0)}(A\cos(0) + B\sin(0)) + 2e^{-2(0)}$

$1 = A + 2$

$A = -1$

$-2 = A(-2e^{-2(0)}\cos(0) - e^{-2(0)}\sin(0)) + B(-2e^{-2(0)}\sin(0) + e^{-2(0)}\cos(0)) - 4e^{-2(0)}$

$-2 = -2A + B - 4$; $-2 = +2 + B$

$\therefore B = 0$

$y_s = e^{-2x}(-\cos x) + 2e^{-2x}$

7 $3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 2x - 3$

$3y'' - 2y' - y = 2x - 3$ .... ①

$3y'' - 2y' - y = 0$ ...... ②

$y = e^{kx}$ $y' = ke^{kx}$ $y'' = k^2e^{kx}$ subst into eqn ②

$3k^2e^{kx} - 2ke^{kx} - e^{kx} = 0$

$e^{kx}(3k^2 - 2k - 1) = 0$

$(3k+1)(k-1) = 0$

$k_1 = -\frac{1}{3}$, $k_2 = 1$

$y_1 = e^{k_1x} = e^{-\frac{1}{3}x}$

$y_2 = e^{k_2x} = e^{x}$

$y_h = c_1y_1 + c_2y_2 = c_1e^{-\frac{1}{3}x} + c_2e^{x}$

let $y_p = Ax^2 + Bx + c$ $y_p' = 2Ax + B$ $y_p'' = 2A$

$3(2A) - 2(2Ax + B) - (Ax^2 + Bx + c) = 2x - 3$

$6A - 4Ax - 2B - Ax^2 - Bx - C = 2x - 3$

$A = 0$

$-4A - B = 2$; $B = -2$

$6A - 2B - C = -3$; $C = 7$

$y_p = -2x + 7$

$y_s = y_h + y_p$

$y_s = c_1e^{-\frac{1}{3}x} + c_2e^{\cancel{+\frac{1}{3}}x} - 2x + 7$

8. $\frac{d^2y}{dx^2} - \frac{6dy}{dx} + 8y = 8e^{4x}$

$y'' - 6y' + 8y = 8e^{4x}$ ..... ①

$y'' - 6y' + 8y = 0$ .... ②

let $y = e^{kx}$ $y' = ke^{kx}$ $y'' = k^2e^{kx}$

$k^2e^{kx} - 6ke^{kx} + 8e^{kx} = 0$

$e^{kx}(k^2 - 6k + 8) = 0$

$k_1 = 2$ $k_2 = 4$

$y_1 = e^{k_1x} = e^{2x}$

$y_2 = e^{k_2x} = e^{4x}$

$y_h = c_1e^{2x} + c_2e^{4x}$

$y_p' = Axe^{4x}$

$y_p' = 4Axe^{4x} + Ae^{4x}$

$y_p'' = 8Ae^{4x} + 16Axe^{4x}$

$8Ae^{4x} + 16xAe^{4x} - 6(Ae^{4x} + 4xe^{4x}) + 8Axe^{4x} = 8e^{4x}$

$8A\cancel{e^{4x}} + 16Ax\cancel{e^{4x}} - 16A\cancel{e^{4x}} - 24Ax\cancel{e^{4x}} + 8Ax\cancel{e^{4x}} = 8\cancel{e^{4x}}$

$8A + \cancel{16Ax} - 6A - \cancel{24Ax} + \cancel{8Ax} = 8$

$2A = 8$

$A = 4$

$y_p = 4Axe^{4x}$

$y_s = y_h + y_p$

$y_s = C_1e^{2x} + C_2e^{4x} + 4Axe^{4x}$