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ENG281 - Engineering maths
Assignment.

i) Parametric equation of a curve is given as eqn 1 & 2

$x = \cos t + t\sin t$ ... eqn 1

$y = \sin t - t\cos t$ ... eqn 2

ii) determin radius R

determine coordinates (h, k) at center of curvature.

Answer

$\overset{u}{x} = \overset{u}{\cos t} + \overset{v}{t\sin t}$

$\frac{dx}{dt} = \frac{du}{dt} + \frac{dv}{dt}$

$= -\sin t +$

$\frac{dv}{dt}$ .. using product rule $t = p$ $\sin t = q$

$\frac{dv}{dt} = p\frac{dq}{dt} + q\frac{dp}{dt}$ $\qquad dp/dt = 1 \quad dq/dt = \cos t$

$= t \cdot \cos t + \sin t \cdot 1$

$= t\cos t + \sin t$

$\frac{dv}{dt} = \sin t + t\cos t.$

$\frac{dx}{dt} = -\cancel{\sin t} + \cancel{\sin t} + t\cos t$

$\frac{dx}{dt} = t\cos t.$

$\overset{y}{y} = \overset{u}{\sin t} - \overset{v}{t\cos t}$

$\frac{dy}{dt} = \frac{du}{dt} - \frac{dv}{dt}$

$v = t\cos t$ let $p = t$ $q = \cos t$ $\quad ; dq/dt = -\sin t \quad dp/dt = 1$

$\frac{dv}{dt} = p\frac{dq}{dt} + q\frac{dp}{dt}$

$= t \cdot -\sin t + \cos t \cdot 1$

$dv/dt = -t\sin t + \cos t$

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$$\frac{dy}{dt} = \cos t - (t\sin t + \cos t) = \cancel{\cos t} + t\sin t - \cancel{\cos t}$$

$$= t\sin t$$

$$\frac{dy}{dt} = t\sin t.$$

$$\frac{dy}{dx} = \frac{dy}{dt} * \frac{dt}{dx}$$

but $\frac{dt}{dx} = \left(\frac{dx}{dt}\right)^{-1}$

$$\frac{dy}{dx} = t\sin t * (t\cos t)^{-1}$$

$$\frac{dy}{dx} = \frac{\cancel{t}\sin t}{\cancel{t}\cos t} = \tan t$$

$$\frac{dy}{dx} = \tan t = \tan\theta$$

$$\theta = \tan^{-1}\tan t$$

$$\theta = t$$

$$\frac{d^2y}{dx^2} = \frac{d\left(\frac{dy}{dx}\right)}{dt} \div \frac{dx}{dt}$$

$$= \sec^2 t \div t\cos t$$

$$= \frac{\sec^2 t}{t\cos t} = \frac{1}{t\cos^3 t}$$

$$\cancel{R = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}} \qquad \cancel{\frac{\left(1+(\tan t)^2\right)^{3/2}}{\frac{1}{t\cos^3 t}}}$$

$$\cancel{= \frac{(1+\tan^2 t)^{3/2}}{\frac{1}{t}\sec^3 t}}$$

$$R = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{d^2y/dx^2} = \frac{\left(1+\left(\frac{\sin t}{\cos t}\right)^2\right)^{3/2}}{\frac{1}{t\cos^3 t}} = \frac{\left(1+\frac{\sin^2 t}{\cos^2 t}\right)^{3/2}}{\frac{1}{t\cos^3 t}}$$

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$h = x - R\sin\theta$

$k = y + R\cos\theta$

$$R = \frac{\left(1 + \frac{\sin^2 t}{\cos^2 t}\right)^{3/2}}{1/t\cos^3 t} = \frac{\left(\frac{\cos^2 t}{\cos^2 t} + \frac{\sin^2 t}{\cos^2 t}\right)^{3/2}}{1/t\cos^3 t} = \frac{\left(\frac{\cos^2 t + \sin^2 t}{\cos^2 t}\right)^{3/2}}{1/t\cos^3 t}$$

$$\frac{\left(\frac{1}{\cos^2 t}\right)^{3/2}}{1/t\cos^3 t}$$

$$\left[\frac{1}{(\cos t)^2}\right]^{3/2} \div \frac{1}{t(\cos t)^3}$$

$$\frac{1^{3/2}}{(\cos t)^{2\times 3/2}} \times \frac{t(\cos t)^3}{1}$$

$$\frac{1}{\cancel{(\cos t)^3}} \times \frac{t\,\cancel{(\cos t)^3}}{1} = t$$

$R = t.$

Recall θ is also $= t$; $R = t$.

(h, k) $x \sim R$

$h = x - R\sin\theta$

$k = y + R\cos\theta$

$x = \cos t + t\sin t$

$y = \sin t - t\cos t$

$h = \cos t + t\sin t - (t\sin t)$

$k = \sin t - t\cos t + (t\cos t)$

$h = \cos t + \cancel{t\sin t} - \cancel{t\sin t} = \cos t$

$k = \sin t - \cancel{t\cos t} + \cancel{t\cos t} = \sin t$

$(h, k) = (\cos t, \sin t)$