NAME: ABAI OLAWALE DAVID

DEPT: CHEMICAL AND PETROLEUM ENGINEERING

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COURSE CODE: CHE 531

COURSE TITLE: PROCESS DYNAMICS AND CONTROL (I)

ANSWERS TO ASSIGNMENT (I)

Let $ \frac{d^{2}y}{dt^{2}}=y^{″}$ and $\frac{dy}{dt}=y'$

Substituting the above equation, we have

$$\left\{y^{″}\right\}-3\left\{y^{'}\right\}+2\left\{y\right\}=2\left\{e^{3t}\right\}$$

Taking the Laplace of both side, we have

$$L\left\{y^{″}\right\}-3L\left\{y^{'}\right\}+2L\left\{y\right\}=2L\left\{e^{3t}\right\}$$

$$s^{2}y\left(s\right)-sy\left(0\right)-y^{'}\left(0\right)-3\left[sy\left(s\right)-y\left(0\right)\right]+2y\left(s\right)=2\left\{\frac{1}{s-3}\right\}$$

$$y\left(0\right)=5, y^{'}(0)=7$$

$$s^{2}y\left(s\right)-5s-7-3sy\left(s\right)+3\left(5\right)+2y\left(s\right)=\frac{2}{s-3}$$

$$s^{2}y\left(s\right)-3sy\left(s\right)+2y\left(s\right)-5s+15-7=\frac{2}{s-3}$$

$$s^{2}y\left(s\right)-3sy\left(s\right)+2y\left(s\right)-5s+8=\frac{2}{s-3}$$

$$y\left(s\right)[s^{2}-3s+2]-5s+8=\frac{2}{s-3}$$

$$y\left(s\right)\left[s^{2}-3s+2\right]=\frac{2}{s-3}-8+5s$$

$$y\left(s\right)\left[s^{2}-3s+2\right]=\frac{2-8\left(s-3\right)+5s(s-3)}{s-3}$$

$$y\left(s\right)\left[s^{2}-3s+2\right]=\frac{2-8s+24+5s^{2}-15s}{s-3}$$

$$y\left(s\right)\left[s^{2}-3s+2\right]=\frac{5s^{2}-23s+26}{s-3}$$

$$y\left(s\right)=\frac{5s^{2}-23s+26}{\left(s-3\right)}×\frac{1}{(s^{2}-3s+2)}$$

Finding he roots of the quadratic equations above, we have

$$y\left(s\right)=\frac{(s-2)×(s-^{13}/\_{5})}{(s-3)×(s-1)×(s-2)}$$

$$y\left(s\right)=\frac{(s-^{13}/\_{5})}{(s-3)×(s-1)}$$

Multiplying the numerator by 5 to remove fraction, we have

$$y\left(s\right)=\frac{5s-13}{(s-3)(s-1)}$$

$$\frac{5s-13}{(s-3)(s-1)}=\frac{A}{s-3}+\frac{B}{s-1}$$

$$\frac{5s-13}{\left(s-3\right)\left(s-1\right)}=\frac{A\left(s-1\right)+B\left(s-3\right)}{\left(s-3\right)\left(s-1\right)}$$

$$5s-13=A\left(s-1\right)+B(s-3)$$

To find the value of (A) let the value of s be 3

$$(5×3)-13=A\left(3-1\right)+B(3-3)$$

$$15-13=A\left(2\right)+B(0)$$

$$2=2A$$

$$A=1$$

To find the value of B let the value of s be 1

$$\left(5×1\right)-13=A\left(1-1\right)+B\left(1-3\right)$$

$$5-13=A\left(0\right)+B\left(-2\right)$$

$$-8=-2B$$

$$B=4$$

$$y\left(s\right)=\frac{5s-13}{(s-3)(s-1)}=\frac{1}{s-3}+\frac{4}{s-1}$$

$$y\left(t\right)=L^{-1}\left\{y\left(s\right)\right\}=L^{-1}\left\{\frac{1}{\left(s-3\right)}+\frac{4}{\left(s-1\right)}\right\}$$

$$y\left(t\right)=e^{3t}+4e^{t}$$