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Dept: Computer Engineering

$$1) \frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$$

$$m^2 - m - 2 = 0$$

$$m^2 - 2m + m - 2 = 0$$

$$(m^2 - 2m)(m - 2) = 0$$

$$m(m - 2) + 1(m - 2) = 0$$

$$(m + 1)(m - 2) = 0$$

$$m_1 = -1 \text{ \& } m_2 = 2$$

$$C.F = y = Ae^{-x} + Be^{2x}$$

$$P.I: y = C$$

$$\frac{dy}{dx} = 0$$

$$\frac{d^2y}{dx^2} = 0$$

$$+2C = 8$$

$$C = -4$$

$$P.I: y = -4$$

$$G.S: y = Ae^{-x} + Be^{2x} - 4$$

$$2) \frac{d^2y}{dx^2} - 4y = 10e^{3x}$$

$$m^2 - 4 = 0$$

$$m^2 = 4$$

$$m = \pm 2$$

$$C.F = y = Ae^{2x} + Be^{-2x}$$

$$P.I: y = Ce^{3x}$$

$$\frac{dy}{dx} = 3Ce^{3x}$$

$$\frac{d^2y}{dx^2} = 9Ce^{3x}$$

$$9Ce^{3x} - 4(Ce^{3x}) = 10e^{3x}$$

$$9Ce^{3x} - 4Ce^{3x} = 10e^{3x}$$

$$9C - 4C = 10$$

$$5C = 10$$

$$C = 2$$

$$P.I = y = 2e^{3x}$$

$$G.S: Ae^{2x} + Be^{-2x} + 2e^{3x}$$

$$3) \frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}$$

$$m^2 + 2m + 1 = 0$$

$$m^2 + m + m + 1 = 0$$

$$m(m+1) + 1(m+1) = 0$$

$$(m+1)(m+1) = 0$$

$$m = -1$$

$$C.F: y = e^{-x}(A+Bx)$$

$$P.I = y = Ce^{-2x}$$

$$\frac{dy}{dx} = -2Ce^{-2x}$$

$$\frac{d^2y}{dx^2} = 4Ce^{-2x}$$

$$4Ce^{-2x} - 4Ce^{-2x} + Ce^{-2x} = e^{-2x}$$

$$4C - 4C + C = 1$$

$$C = 1$$

$$P.I: y = e^{-2x}$$

$$G.S = e^{-x}(A+Bx) + e^{-2x}$$

$$4) \frac{d^2y}{dx^2} + 25y = 5x^2 + x$$

$$m^2 + 25 = 0$$

$$m^2 = -25$$

$$m = \pm j5$$

$$C.F: y = A\cos 5x + B\sin 5x$$

$$P.I: y = Cx^2 + Dx + E + Fx + G$$

$$\frac{dy}{dx} = 2Cx + D + F$$

$$\frac{d^2y}{dx^2} = 2C$$

$$2C + 25Cx^2 + 25Dx + 25E + 25Fx + 25G = 5x^2 + x$$

$$25Cx^2 + (25D + 25F)x + (2C + 25E + 25G) = 5x^2 + x$$

$$25C = 5$$

$$C = 5$$

$$25D + 25F = 1 \dots \textcircled{1}$$

$$D + F = \frac{1}{25}$$

$$2C + 25E + 25G = 0$$

$$10 + 25E + 25G = 0$$

$$E + G = -\frac{2}{5}$$

$$P.I: y = 5x^2 + \frac{1}{25}x + \frac{-2}{5}$$

$$G.S = A \cos 5x + B \sin 5x + 5x^2 + \frac{1}{25}x - \frac{2}{5}$$

$$5) \frac{d^2y}{dx^2} - 2 \frac{dy}{dx} + y = 4 \sin x$$

$$m^2 - 2m + 1 = 0$$

$$m^2 - m - m + 1 = 0$$

$$m(m-1) - 1(m-1) = 0$$

$$(m-1)(m-1) = 0$$

$$m = 1$$

$$C.F: y = e^x(A + Bx)$$

$$P.I: y = C \cos x + D \sin x$$

$$\frac{dy}{dx} = -C \sin x + D \cos x$$

$$\frac{d^2y}{dx^2} = -C \cos x - D \sin x$$

$$-C \cos x - D \sin x + 2C \sin x - 2D \cos x + C \cos x + D \sin x = 4 \sin x$$

$$\cos x (-C - 2D + C) + \sin x (-D + 2C + D) = 4 \sin x$$

$$\cos x (-2D) + \sin x (2C) = 4 \sin x$$

$$-2D = 0$$

$$D = 0$$

$$2C = 4$$

$$C = 2$$

$$P.I: y = 2 \cos x$$

$$G.S = y = e^x(A + Bx) + 2 \cos x$$

b) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$ given that at $x=0$, $y=1$ and $\frac{dy}{dx} = -2$

$$m^2 + 4m + 5 = 0$$

$$a=1, b=4, c=5$$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-4 \pm \sqrt{16 - 4(1)(5)}}{2}$$

$$= \frac{-4 \pm \sqrt{-4}}{2}$$

$$= \frac{-4 \pm \sqrt{-1} \sqrt{4}}{2}$$

$$= \frac{-4 \pm \sqrt{1} \cdot 2}{2}$$

$$= -2 \pm j$$

$$\alpha = -2, \beta = 1$$

$$y = e^{-2x} (A \cos x + B \sin x)$$

PI: $y = Ce^{-2x}$

$$\frac{dy}{dx} = -2Ce^{-2x}$$

$$\frac{d^2y}{dx^2} = 4Ce^{-2x}$$

$$4Ce^{-2x} - 8Ce^{-2x} + 5Ce^{-2x} = 2e^{-2x}$$

$$4C - 8C + 5C = 2$$

$$C = 2$$

P.I: $y = 2e^{-2x}$

G.S $\Rightarrow y = e^{-2x} (A \cos x + B \sin x) + 2e^{-2x}$

at $x=0$, $y=1$ and $\frac{dy}{dx} = -2$

$$1 = e^{-2(0)} (A \cos 0 + B \sin 0) + 2e^{-2(0)}$$

$$1 = A + 2$$

$$A = -1$$

$$\frac{dy}{dx} = e^{-2x}(-A \sin x + B \cos x) - 2e^{-2x}(A \cos x + B \sin x) - 4e^{-2x}$$

$$-2 = e^{-2(0)}(-A \sin 0 + B \cos 0) - 2e^{-2(0)}(A \cos 0 + B \sin 0) - 4e^{-2(0)}$$

$$-2 = 1(B) - A - 4$$

$$-2 = B - A - 4$$

$$2 = B - A$$

$$2 = B + 1$$

$$B = 1$$

$$y = e^{-2x}(-1 \cos x + \sin x) + 2e^{-2x}$$

$$7) \cdot 3 \frac{d^2y}{dx^2} - 2 \frac{dy}{dx} - y = 2x - 3$$

$$3m^2 - 2m - 1 = 0$$

$$3m^2 + m - 3m - 1 = 0$$

$$m(3m+1) - 1(3m+1) = 0$$

$$(m-1)(3m+1) = 0$$

$$m_1 = 1 \text{ \& } m_2 = -\frac{1}{3}$$

$$\text{C.F. : } y = Ae^x + Be^{-\frac{x}{3}}$$

$$\text{P.I. : } y = Cx + D - E$$

$$\frac{dy}{dx} = C$$

$$\frac{d^2y}{dx^2} = 0$$

$$-2C - Cx + D - E = 2x - 3$$

$$-Cx + (C + D - E) = 2x - 3$$

$$2 = -C$$

$$C = -2$$

$$C + D - E = -3$$

$$-2 + D - E = -3$$

$$D - E = -1$$

$$\text{P.I. } y = -2x - 1$$

$$\text{C.S. } y = Ae^{2x} + Be^{-\frac{2}{3}x} - 2x - 1$$

$$8) \frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$$

$$m^2 - 6m + 8 = 0$$

$$m^2 - 2m - 4m + 8 = 0$$

$$m(m-2) - 4(m-2) = 0$$

$$m_1 = 4 \quad m_2 = 2$$

$$\text{C.F. } y = Ae^{4x} + Be^{2x}$$

$$\text{P.I. } y = Cxe^{4x}$$

$$\frac{dy}{dx} = Cx(4e^{4x}) + Ce^{4x}$$

$$\frac{d^2y}{dx^2} = Cx(16e^{4x}) + C(4e^{4x}) + 4e^{4x}$$

$$Cx(16e^{4x}) + C(4e^{4x}) + (4e^{4x} - 6Cx(4e^{4x}) - 6Ce^{4x} + 8Cxe^{4x}) = 8e^{4x}$$

$$16Cx + 4C + 4C - 24Cx - 6C + 8Cx = 8$$

$$16Cx + 2C - 24Cx + 8Cx = 8$$

$$2C = 8$$

$$C = 4$$

$$\text{P.I. } y = 4xe^{4x}$$

$$\text{C.S. } y = Ae^{4x} + Be^{2x} + 4xe^{4x}$$