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ENG 281

1) Parametric equation of a curve is given as eqn 1 & 2.

$x = \cos t + \sin t$ ---- eqn 1.

$y = \sin t - t\cos t$ ---- eqn (2).

determine radius R.

determine coordinates (h, k) at centre of curvature.

Soln

$x = \cos t + \sin t$

$\frac{dx}{dt} = \frac{du}{dt} + \frac{dv}{dt}$

$= -\sin t$.

du/dt - using product rule $t = p$, $\sin t = q$

$\frac{du}{dt} = p\frac{dq}{dt} + q\frac{dp}{dt}$.

$= t\cos t + \sin t \cdot 1$

$= t\cos t + \sin t$.

$\frac{du}{dt} = \sin t + \cos t$.

$\frac{dx}{dt} = \sin t + t\cos t$.

$\frac{dx}{dt} = -\sin t + \sin t + t\cos t$

$\frac{dx}{dt} = t\cos t$.

$y = \sin t - t\cos t$.

$\frac{dy}{dt} = \frac{du}{dt} - \frac{dw}{dt}$.

$v = t\cos t$, i.e. $I = \cos t$.

$dq/dt = -\sin t$ $dp/dt = 1$

$\frac{dw}{dt} = p\frac{dq}{dt} + q\frac{dp}{dt}$

$\frac{dw}{dt} = t \cdot -\sin t + \cos t \cdot 1$

$\frac{dw}{dt} = -t\sin t + \cos t$.

$\frac{dy}{dt} = \cos t - (t\sin t + \cos t) = t\sin t$.

$\frac{dy}{dt} = t\sin t$.

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$t\sin t \frac{dt}{dx} = \left(\frac{dx}{dt}\right)^{-1}$

$\frac{dy}{dx} = t\sin t \times (t\cos t)^{-1}$

$\frac{dy}{dx} = \frac{t\sin t}{t\cos t} = \tan t$ $\frac{dy}{dx} = \tan t = \tan\theta$

$\theta = \tan^{-1}\tan t$

$\theta = t$.

$\frac{d^2y}{dx^2} = \frac{d(dy/dx)}{dt} \div \frac{dx}{dt}$

$= \sec^2 t \div t\cos t$.

$= \frac{\sec^2 t}{t\cos t} = \frac{1}{t\cos^3 t}$.

$R = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} = \frac{\left[1 + \left(\frac{\sin^2 t}{\cos t}\right)^2\right]^{3/2}}{\frac{1}{t\cos^3 t}}$.

$= \frac{\left(1 + \frac{\sin^2 t}{\cos^2 t}\right)^{3/2}}{\frac{1}{t\cos^3 t}} = \frac{\left(\frac{\cos^2 t + \sin^2 t}{\cos^2 t}\right)^{3/2}}{\frac{1}{t\cos^3 t}}$

$$= \left(\frac{\cos^2 t + \sin^2 t}{\cos^2 t}\right)^{3/2} \cdot \frac{1}{t\cos^3 t}$$

$$\left[\frac{1}{(\cos t)^2}\right]^{3/2} \cdot \frac{1}{t\cos^3 t}$$

$$\left[\frac{1}{(\cos t)^2}\right]^{3/2} \div \frac{1}{t(\cos t)^3}$$

$$\frac{1^{3/2}}{(\cos t)^{2\times 3/2}} \times \frac{t(\cos t)^3}{1}$$

$$\frac{1}{(\cos t)^3} \times \frac{t(\cos t)^3}{1} = t.$$

$R = t.$

Recall θ is also $= t$; $R = t$.

(h, k)

$h = x - R\sin\theta$

$k = y + R\cos\theta$

$x = \cos t + t\sin t$

$y = \sin t - t\cos t$

$h = \cos t + t\sin t - (t\sin t)$

$k = \sin t - t\cos t + (t\cos t)$

$h = \cos t + t\sin t - t\sin h = \cos t$

$k = \sin t - t\cos t + t\cos t = \sin t$

$(h, k) = (\cos t, \sin t)$