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15/ENG03/009
300L

Assignment

1) $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$

$m^2 - m - 2 = 0$, $m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$m = \frac{+1 \pm \sqrt{(-1)^2 - 4(1)(-2)}}{2(1)}$

$m = \frac{1 \pm \sqrt{9}}{2}$ = $m_1 = 2$, $m_2 = -1$

$y = Ae^{mx} + Be^{m_2 x}$

$C.F = Ae^{2x} + Be^{-x}$

$P.I = y = k$, $dy/dx = 0$ $\frac{d^2y}{dx^2} = 0$

$0 - 0 - 2(c) = 8$

$-2c = 8$

$c = -4$

$P.I : y = 4$

$G.S = C.F + P.I$

$G.S = y = Ae^{2x} + Be^{-x} - 4$

2) $\frac{d^2y}{dx^2} - 4y = 5e^{3x}$

$m^2 - 4 = 0$

$m^2 = 4$

$m_1 = 2$, $m_2 = -2$

$C.F : y = A\cosh 2x + B\sin 2x$

$P.I = y = ce^{3x}$

$\frac{dy}{dx} = 3ce^{3x}$, $\frac{d^2y}{dx^2} = 9ce^{3x}$

$9ce^{3x} - 4(ce^{3x}) = 10e^{3x}$

$9c - 4c = 0$

$5c = 10$, $c = 2e^{3x}$; $P.I = y = 2e^{3x}$

$G.S : y = A\cosh 2x + B\sinh 2x + 2e^{3x}$

3) $\frac{d^2y}{dx^2} + \frac{2dy}{dx} + y = e^{-2x}$

$m^2 + 2m + 1 = 0$

$(m+1)(m+1)$

$m = -1$ Twice

C.F = $y = e^{-x}(A+Bx)$

P.I, $y = ce^{-2x}$

$\frac{dy}{dx} = -2ce^{-2x}$, $\frac{d^2y}{dx^2} = 4ce^{-2x}$

$4ce^{-2x} + 2(-2ce^{-2x}) + ce^{-2x} = e^{-2x}$

$4c - 4c + c = 1$

$c = 1$

P.I : $y = 1e^{-2x}$

G.S = $y = e^{-x}(A+Bx) + e^{-2x}$

4) $\frac{d^2y}{dx^2} + 25y = 5x^2 + x$

$m^2 + 25 = 0$

$m^2 = -25$

$m = 5j$

C.F : $y = e^{0x}(A\cos 5x + B\sin 5x)$

P.I : $y = Cx^2 + Dx + E$

$\frac{dy}{dx} = 2Cx + D$, $\frac{d^2y}{dx^2} = 2C$

$2C + 25Cx^2 + 25Dx + 25E = 5x^2 + x$

$2C + 25E = 0$

$C = 5/25 = 1/5$

$D = 1/25$ $E = -2/5 \times 1/25 = -2/125$

G.S : $y = e^{0x}(A\cos 5x + B\sin 5x) + 1/5x^2 + 1/25x - \frac{2}{125}$

$y = A\cos 5x + B\sin 5x + (x^2 + 1/5x - 2/25)\ 1/2$

c) $\frac{d^2y}{dx^2} - \frac{2dy}{dx} + y = 4\sin x$

$m^2 - 2m + 1 = 0$

$(x-1)(x-1)$

$x = 1$ Twice

$C.F = y = e^x(A + Bx)$

$P.I = y = C\cos x + D\sin x$

$dy/dx = -C\sin x + D\cos x$; $d^2y/dx^2 = -C\cos x - D\sin x$

$-C\cos x - D\sin x + 2C\sin x - 2D\cos x + C\cos x + D\sin x = 4\sin x$

$\cos x(-C - 2D + C) + \sin x(D - 2C + D) = 4\sin x$

$-2D = 0$

$x = 4 \quad C = 2$

$P.I = y = 2\cos x + 0$; $G.S: y = e^x(A + Bx) + 2\cos x$

d) $\frac{d^2y}{dx^2} + \frac{4dy}{dx} + 5y = 2e^{-2x}$ when $x = 0$, $y = 1$ and $dy/dx = -2$

$m^2 + 4m + 5 = 0$

$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$m = \frac{-4 \pm \sqrt{-4}}{2}$, $m = \frac{-4 \pm 2i}{2} = -2 \pm i$

$\alpha = -2$, $\beta = 1$

$C.F = y = e^{-2x}(A\cos ix + B\sin ix)$

$P.I = y = Ce^{-2x}$

$\frac{dy}{dx} = -2Ce^{-2x}$, $\frac{d^2y}{dx^2} = 4Ce^{-2x}$

$4Ce^{-2x} - 8Ce^{-2x} + 5Ce^{-2x} = 2$

$C = 2$

$P.I = 2e^{-2x}$

$G.S: y = e^{-2x}(A\cos x + B\sin x) + 2e^{-2x}$

$\frac{dy}{dx} = e^{-2x}(-A\sin x + B\cos x) + (-2e^{-2x}(A\cos x + B\sin x)) + (-4e^{-2x})$

$\frac{dy}{dx} = e^{-2x}(-A\sin x + B\cos x) - 2e^{-2x}(A\cos x + B\sin x) - 4e^{-2x}$

where $x=0$, $y=1$ & $dy/dx = -2$

$1 = e^{-2(0)}(A\cos(0) + B\sin(0)) + 2e^{-2(0)}$

$1 = 1(A) + 2$

$A = -1$

$-2 = e^{-2(0)}(-A\sin(0) + B\cos(0)) + 2e^{-2(0)}$

$1 = 1(A) + 2$

$A = 1$

$-2 = e^{-2(0)}(-A\sin(0) + B\cos(0)) - 2e^{-2(0)}(A\cos 0 + B\sin 0) - 4e^{-0}$

$-2 = 1(B) - 1(A) - 4 \quad = \quad -2 = B - 1 - 4$

$B = 5$

G.S : $y = e^{-2x}(-\cos 2x - 5\sin 2x) + 2e^{-2x}$ //

7) $3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 2x - 3$

$3m^2 - 2m - 1 = 0$

$(m-1)(3m+1)$

$m_1 = 1, \quad m_2 = -1/3$

C.F $= y = Ae^x + Be^{-x/3}$

P.I $= y = cx + D$

$\frac{dy}{dx} = c, \quad \frac{d^2y}{dx^2} = 0$

$3(0) - 2(c) - cx - D = 2x - 3$

$-2c - cx - D = 2x - 3$

$-cx = 2x, \quad c = -2$

$-2c - D = -3$

$-2(-2) - D = 3$

$4 - D = 3$

$D = 7$

P.I $= y = -2x + 7$

G.S $= y = Ae^x + Be^{-x/3} - 2x + 7$ //

8) $\frac{d^2y}{dx^2} - 6\,dy/dx + 8y = 4e^{2x}$

$m^2 - 6m + 8 = 0$

$(m-4)(m-2)$

$m_1 = 4, \quad m_2 = 2$

C.F $= y = Ae^{4x} + Be^{2x}$

P.I $= y = ce^{4x}$

$\frac{dy}{dx} = 4ce^{4x}, \quad \frac{d^2y}{dx^2} = 16ce^{4x}$

$16ce^{4x} - 6(4ce^{4x}) + 8(ce^{4x}) = 8e^{4x}$

$16c - 24c + 8c = 8$

$0c = 8$

$c = 0$

G.S $= y = Ae^{4x} + Be^{2x}$ //