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1. $x = \cos t + t\sin t$

$y = \sin t - t\cos t$

(a) An expression for the radius of curvature (R) in terms of t

Soln

$x = \cos t + \sin t$

$dx/dt = -\sin t + t\cos t + \sin t$

$= t\cos t$

$y = \sin t - t\cos t$

$dy/dt = \cos t + t\sin t - \cos t$

$= t\sin t$

$dy/dx = dy/dt \times dt/dx = \frac{t\cos t}{1} \times \frac{1}{t\sin t} = \frac{t\cos t}{t\sin t}$

$dy/dx = \frac{\cos t}{\sin t}$

$\frac{d}{dx}\left(dy/dx\right) = d^2y/dx^2$

$U = \sin t, \quad v = \cos t$

$du/dt = \cos t, \quad dv/dt = -\sin t$

$d^2y/dx^2 = \frac{V du/dx - U dv/dx}{V^2}$ ~~$\times dy/dx$~~

$= \frac{\cos t(\cos t) - \sin t(-\sin t)}{(\cos t)^2} \times \frac{1}{t\cos t}$

$= \frac{\cos^2 t + \sin^2 t}{\cos^2 t} \times \frac{1}{\cos t}$

$d^2y/dx^2 = \frac{1}{\cos^3 t}$ (where $\cos^2\theta + \sin^2\theta = 1$)

$\therefore R = \left(\frac{1 + (dy/dx)^2}{d^2y/dx^2}\right)^{3/2}$

$$R = \frac{\left(1 + \left[\frac{\sin t}{\cos t}\right]^2\right)^{3/2}}{\frac{1}{t\cos^3 t}}$$

$$R = \frac{\left(1 + \frac{\sin^2 t}{\cos^2 t}\right)^{3/2}}{\frac{1}{t\cos^3 t}}$$

$$R = \frac{\left(\frac{\cos^2 t + \sin^2 t}{\cos^2 t}\right)^{3/2}}{\frac{1}{t\cos^3 t}}$$

But $\cos^2\theta + \sin^2 t = 1$

$$R = \left(\frac{1}{\cos^2 t}\right)^{3/2} \div \frac{1}{t\cos^3 t}$$

$$R = \frac{1}{\cos^3 t} \times \frac{t\cos^3 t}{1}$$

$$R = t$$

b. Expressions for the coordinates (h, k) or the centre of Curvature in terms of t

Soln

$h = x_1 - R\sin\theta$

~~$k = x_1 + R\cos\theta$~~

$k = y_1 + R\cos\theta$

Where $R = t$, $\theta = \tan^{-1}(dy/dx)$, $dy/dx = \sin t / \cos t = \tan t$, $\theta = t$

$\Rightarrow h = x_1 - t\sin t$

$k = y_1 + t\cos t$

When, $x = \cos t + t\sin t$

$y = \sin t - t\cos t$

$x = x_1$, $y = y_1$

$\therefore \quad h = \cos t + t \sin t - t \sin t$

$h = \underline{\underline{\cos t}}$

$K = \sin t - t\cos t + t\cos t$

$K = \underline{\underline{\sin t}}$

$(h, K) = [\cos t, \sin t]$