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DEPT: MECHANICAL ENGINEERING
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COURSE: ENG 281 (ENGINEERING MATHEMATICS

1. Evaluate the following limits of function

(a) $\lim_{x\to\pi/2}\left\{\frac{(x^2-\pi/4)\sin(\cos x)}{x-\pi/2}\right\}$ (b) $\lim_{x\to\pi/2} \ln\left(\frac{\exp(3x^2+2x-1)}{x+1}\right)$

(c) $\lim_{x\to 2\sqrt{3}} \cos\left\{\frac{\sin^{-1}(x-2)}{(x-\sqrt{3})}\right\}$ (d) $\lim_{x\to 4}\left\{\frac{x^2-8x+16}{x^2-5x+4}\right\}$

2. Determine whether each of the following series is convergent:

(a) $\frac{2}{2\times3}+\frac{2}{3\times4}+\frac{2}{4\times5}+\frac{2}{5\times6}+\ldots$

(b) $\frac{2}{1^2}+\frac{2}{2^2}+\frac{2}{3^2}+\frac{2}{4^2}+\ldots$

(c) $U_n=\frac{1+2n^2}{1+n^2}$

3. Find the range of values of x for which the series below is absolutely convergent:

$$\frac{x}{27}+\frac{x^2}{125}+\ldots+\frac{x^n}{(2n+1)^3}$$

4. Evaluate using L'Hopital Rule;

$$\lim_{x\to0}\left(\frac{\sin x-\cos x}{x^3}\right)$$

Solution

1 (a) $\lim_{x \to \pi/2} \left\{ \frac{(x^2 - \pi/4)\sin(\cos x)}{x - \pi/2} \right\}$

$dy/dx = \frac{(2x - 0)(-\sin^2 x)}{1 - 0}$, $y = \left[\frac{(x^2 - \pi/4)\sin(\cos x)}{x - \pi/2}\right]$

Direct Substing $x \longrightarrow \pi/2$

$= 2\left[\pi/2\right]\left(-\sin^2(\pi/2)\right)$

$= -\pi(-1)^2$

$= -\pi$

$\lim_{x \to \pi/2} \left[\frac{(x^2 - \pi/4)\sin(\cos x)}{x - \pi/2}\right] = -\pi$

(b) $\lim_{x \to -1} \ln\left[\exp\left(\frac{3x^2 + 2x - 1}{x + 1}\right)\right]$

$\ln\left[\exp\left(\frac{(3x - 1)\cancel{(x+1)}}{\cancel{x+1}}\right)\right]$

$\lim_{x \to -1} \ln\left[\exp\left[3(-1) - 1\right]\right]$

$\ln\left(\exp(-3 - 1)\right) = -4$

(c) $\lim_{x \to 2+\sqrt{3}} \cos\left[\sin^{-1}\left(\frac{x - 2}{x - \sqrt{3}}\right)\right]$

$\cos\left[\sin^{-1}\left(\frac{\cancel{2} + \sqrt{3} - \cancel{2}}{2 + \cancel{\sqrt{3}} - \cancel{\sqrt{3}}}\right)\right] \Rightarrow \cos\left[\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right]$

$\Rightarrow \cos 60 = 1/2$

(d) $\lim_{x \to 4} \left[\frac{x^2 - 8x + 16}{x^2 - 5x + 4}\right]$

$\lim_{x \to 4} \left[\frac{\cancel{(x-4)}(x+4)}{(x-1)\cancel{(x-4)}}\right]$

$\lim_{x \to 4} \left[\frac{x + \cancel{4}}{\cancel{x+1}}\right] \left[\frac{x + 4}{x - 1}\right]$

$= \frac{4 - 4}{\cancel{4} - 1} = 0/3$ (Undefined)

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2. (a) $\frac{2}{2\times3} + \frac{2}{3\times4} + \frac{2}{4\times5} + \frac{2}{5\times6} \dots, \frac{2}{(n+1)(n+2)}$

$U_n = \frac{2}{(n+1)(n+2)}, \quad U_{n+1} = \frac{2}{(n+2)(n+3)}$

$\lim_{n\to\infty} \frac{U_{n+1}}{U_n} = \frac{2}{(n+2)(n+3)} \times \frac{(n+1)(n+2)}{2} \Rightarrow \frac{2(n^2+3n+2)}{2(n^2+5n+6)} \Rightarrow$ ~~$2n+$~~

$\frac{n^2/n^2 + 3n/n^2 + 2/n^2}{n^2/n^2 + 5n/n^2 + 6/n^2} \Rightarrow \frac{1 + 3/n + 2/n^2}{1 + 5/n + 6/n^2}$

$n \to \infty, \quad \frac{1 + 3/0 + 2/0}{1 + 5/0 + 6/0} = \frac{1}{1} = 1$

$\therefore \frac{U_{n+1}}{U_n} = 1$, Series is divergent or Convergent

Further test

$\lim_{n\to\infty} U_n = \frac{2}{(n+1)(n+2)} \Rightarrow \frac{2}{(0+1)(0+2)} = \frac{2}{2} = 1$

$U_n \neq 0 \therefore$ the series is divergent.

(b) $\frac{2}{1^2} + \frac{2^2}{2^2} + \frac{2^3}{3^2} + \frac{2^4}{4^2} + \dots \frac{2^n}{n^2}$

$U_n = \frac{2^n}{n^2}, \quad U_{n+1} = \frac{2^{n+1}}{(n+1)^2}$

$\lim_{n\to\infty} = \frac{U_{n+1}}{U_n} = \frac{2^{n+1}}{n^2+2n+1} \times \frac{n^2}{2^n} \Rightarrow \frac{2^n \times 2 \times n^2}{2^n(n^2+2n+1)}$

$\lim_{n\to\infty} \cdot \frac{2n^2}{n^2+2n+1} = \frac{2n^2/n^2}{n^2/n^2 + 2n/n^2 + 1/n^2} \Rightarrow \frac{2}{1 + 2/n + 1/n^2}$

$n \to \infty, \quad \frac{2}{1+0+0} = 2$

$\therefore \frac{U_{n+1}}{U_n} > 1$, the series is divergent

(c) $U_n = \frac{1+2n^2}{1+n^2}$

$\lim_{n\to\infty} \frac{1+2n^2}{1+n^2} = \frac{1/n^2 + 2}{1/n^2 + 1} = \frac{2+0}{1+0} = \frac{2}{1} = 2$

$n \to 1/n \to 0$

$U_n \neq 0 \therefore$ Series is divergent

3. $\frac{x}{27} + \frac{x^2}{125} + \dots \frac{x^n}{(2n+1)^3}$

$$U_n = \frac{x^n}{(2n+1)^3}, \quad U_{n+1} = \frac{x^{n+1}}{(2n+2)^3}$$

$$\lim \frac{U_{n+1}}{U_n} = \frac{x^{n+1}}{(2n+2)^3} \times \frac{(2n+1)^3}{x^n}$$

$$= \frac{x(2n+1)^3}{(2n+2)^3} \Rightarrow \frac{x[8n^3+12n^2+n+1]}{8n^3+24n^2+24n+8}$$

$$= \frac{x(8n^3/n^3 + 12n^2/n^3 + n/n^3 + 1/n^3)}{(8n^3/n^3 + 24n^2/n^3 + 24n/n^3 + 8/n^3)} \Longrightarrow$$

$$= \frac{x(8 + 12/n + 1/n^2 + 1/n^3)}{(8 + 12/n + 24/n^2 + 8/n^3)} = \cancel{x(8+0+}$$

as $n \to \infty$, $1/n \to 0$

$$\frac{x(8+0+0+0)}{(8+0+0+0)} = \frac{8x}{8} \Rightarrow x = 1$$

$\therefore x \leq 1$

4. $\lim\limits_{x\to 0} \left[\frac{\sin x - \cos x}{x^3}\right]$

By using L'hôpital's rule

$$y = \left[\frac{\sin x - \cos x}{x^3}\right]$$

$$\frac{dy}{dx} = \left[\frac{\cos x + \sin x}{3x^2}\right]$$

$$\frac{d^2y}{dx^2} = \frac{-\sin x + \cos x}{6x}$$

$$\frac{d^3y}{dx^3} = \frac{-\cos x - \sin x}{6}$$

$$\lim_{x\to 0} = \frac{-\cos 0 - \sin 0}{6} = \frac{-1-0}{6} = -1/6$$

$$\lim_{x\to 0} \left[\frac{\sin x - \cos x}{x^3}\right] = -1/6$$