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1) Evaluate the following limits of function:

a) $\lim_{x \to \pi/2} \left(\frac{(x^2 - \pi/4)\sin(\cos x)}{x - \pi/2} \right)$ (b) $\lim_{x \to \pi/2} \ln\left(\exp \frac{(3x^2 + 2x - 1)}{x+1} \right)$

c) $\lim_{x \to 2+\sqrt{3}} \cos\left(\frac{\sin^{-1}(x-2)}{(x-\sqrt{3})} \right)$ (d) $\lim_{x \to 4} \left[\frac{x^2 - 8x + 16}{x^2 - 5x + 4} \right]$

Solution

a) $\lim_{x \to \pi/2} \left(\frac{(x^2 - \pi/4)\sin(\cos x)}{x - \pi/2} \right)$

differentiating: $\frac{(2x - 0)\sin(-\sin x)}{1 - 0}$

$= \lim_{x \to \frac{\pi}{2}} \frac{(2x)\sin^2 x}{1} = 2(\pi/2) - \sin^2\left(\frac{\pi}{2}\right)$

$= -\pi(-1)^2$

$= -\pi$

$\therefore \lim_{x \to \pi/2} \left[\frac{(x^2 - \pi/4)\sin(\cos x)}{x - \pi/2} \right] = -\pi$

b) $\lim_{x \to -1} \ln\left[\exp\left(\frac{3x^2 + 2x - 1}{x+1} \right) \right]$

$\ln\left[\exp\left(\frac{(3x-1)\cancel{(x+1)}}{\cancel{(x+1)}} \right) \right] = \lim_{x \to -1} \ln[\exp(3(-1) - 1)]$

$\lim_{x \to -1} \ln\left[\exp\left(\frac{3x^2 + 2x - 1}{x+1} \right) \right] = \ln(\exp(-3-1)]$

$\therefore \lim_{x \to -1} \ln(\exp(-3-1)] = -4$

c) $\lim_{x \to 2+\sqrt{3}} \cos\left[\sin^{-1}\left(\frac{x-2}{x+\sqrt{3}}\right)\right]$

$\cos\left[\sin^{-1}\left(\frac{2+\sqrt{3}-2}{2+\sqrt{3}-\sqrt{3}}\right)\right] = \cos\left[\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right]$

$\therefore \lim_{x \to 2+\sqrt{3}} \cos\left[\sin^{-1}\left(\frac{x-2}{x+\sqrt{3}}\right)\right] = \cos 60$

$= \frac{1}{2}$

d) $\lim_{x \to 4} \left[\frac{x^2-8x+16}{x^2-5x+4}\right]$

$\lim_{x \to 4} \left[\frac{\cancel{(x-4)}(x+4)}{(x-1)\cancel{(x-4)}}\right] = \lim_{x \to 4} \left[\frac{x+4}{x-1}\right]$

$= \frac{4-4}{4-1}$

$\therefore \lim_{x \to 4} \left[\frac{x^2-8x+16}{x^2-5x+4}\right] = \frac{0}{3}$ (undefined)

2 Determine whether each of the following series is convergent:

a) $\frac{2}{2\times3} + \frac{2}{3\times4} + \frac{2}{4\times5} + \frac{2}{5\times6} + \ldots\ldots$

b) $\frac{2}{1^2} + \frac{2}{2^2} + \frac{2}{3^2} + \frac{2}{4^2} + \ldots\ldots$

c $U_n = \frac{1+2n^2}{1+n^2}$

SOLUTION

a) $\frac{2}{2\times3} + \frac{2}{3\times4} + \frac{2}{4\times5} + \frac{2}{5\times6} + \ldots\ldots +$

$U_n = \frac{2}{(n+1)(n+1)}$ $\quad U_{n+1} = \frac{2}{(n+2)(n+3)}$

Using D'alemberts ratio

$\lim_{n \to \infty} \frac{U_{n+1}}{U_n}$

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$$\frac{U_{n+1}}{U_n} = \frac{2}{\cancel{(n+2)}(n+3)} \times \frac{(n+1)\cancel{(n+2)}}{\cancel{2}} = \frac{(n+1)}{(n+3)}$$

$$\lim_{n\to\infty}\left(\frac{n+1}{n+3}\right) = \lim_{n\to\infty}\left(\frac{1+1/n}{1+3/n}\right)$$

$$= \cancel{\left(\frac{1+0}{1}\right.} \quad \frac{1}{1} = \underline{1}$$

∴ The series is ~~convergente~~ is undetermined using comparison test

$$U_n = \frac{2}{(n+1)(n+2)}$$

$$T_n = \frac{2}{n^2}$$

$$U_n < T_n = \frac{2}{(n+1)(n+2)} < \frac{2}{n^2}$$

Since T_n which is convergent $< U_n$ ∴ U_n is convergent.

b) $\frac{2}{1^2} + \frac{2}{2^2} + \frac{2}{3^2} + \frac{2}{4^2} + \cdots$

The series is convergent since the power on the denominator is greater than one. It is a convergent P-series

c) $U_n = \frac{1+2n^2}{1+n}$ $\qquad U_{n+1} = \frac{1+2(n+1)^2}{1+(n+1)}$

$$U_{n+1} = \frac{1+2[n^2+2n+1]}{2+n}$$

$$\frac{U_{n+1}}{U_n} = \frac{2n^3+4n+3}{2+n} \times \frac{1+n}{1+2n^2}$$

$$\frac{U_{n+1}}{U_n} = \frac{(2n^2+4n+3)(1+n)}{(2+n)(1+2n^2)}$$

$$\frac{U_{n+1}}{U_n} = \frac{(2n^3+6n^2+7n+3)}{(2n^3+4n^2+n+3)}$$

$$\lim_{n\to\infty}\left(\frac{U_{n+1}}{U_n}\right) = \lim_{n\to\infty}\left(\frac{2+6/n+7/n+3/n^3}{2+4/n+1/n+2/n^3}\right)$$

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$$\lim_{n\to\infty}\left(\frac{U_{n+1}}{U_n}\right) = 1$$

$$\lim_{n\to\infty} U_n = \lim_{n\to\infty}\left(\frac{1+2n^2}{1+n}\right)$$

$$= \lim_{n\to\infty}\left(\frac{1/n^2 + 2}{1/n^2 + 1/n}\right)$$

$$\therefore \lim_{n\to\infty} = 1$$

The series is divergent

$\therefore$

4) $$\lim_{x\to 0}\left(\frac{\sin x - \cos x}{x^3}\right)$$

$$\lim_{x\to 0}\left(\frac{\cos x + \sin x}{3x^2}\right) = \frac{-\sin x + \cos x}{6x}$$

$$= \frac{-\cos x - \sin 0}{6} = \frac{-1-0}{6}$$

$$\therefore \lim_{x\to 0}\left(\frac{\sin x - \cos x}{x^3}\right) = \frac{-1}{6}$$

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$$\lim_{n\to\infty}\left(\frac{U_{n+1}}{U_n}\right) = 1$$

$$\lim_{n\to\infty} U_n = \lim_{n\to\infty}\left(\frac{1+2n^2}{1+n}\right)$$

$$= \lim_{n\to\infty}\left(\frac{1/n^2+2}{1/n^2+1/n}\right)$$

$$\therefore \lim_{n\to\infty} = 1$$

The series is divergent

4) $$\lim_{x\to 0}\left(\frac{\sin x - \cos x}{x^3}\right)$$

$$\lim_{x\to 0}\left(\frac{\cos x + \sin x}{3x^2}\right) = \frac{-\sin x + \cos x}{6x}$$

$$= \frac{-\cos x - \sin 0}{6} = \frac{-1-0}{6}$$

$$\therefore \lim_{x\to 0}\left(\frac{\sin x - \cos x}{x^3}\right) = \frac{-1}{6}$$

3) Find the range of values for x for which the series below is absolutely convergent

$$\frac{x}{27} + \frac{x^2}{125} + \cdots + \frac{x^n}{(2n+1)^3}$$

Solution

$$|U_n| = \frac{x^n}{(2n+1)^3}$$

$$|U_{n+1}| = \frac{x^{n+1}}{(2(n+1)+1)^3} = \frac{x^{n+1}}{(2n+3)^3}$$

$$\left|\frac{U_{n+1}}{U_n}\right| = \frac{x^{n+1}}{(2n+3)^3} \cdot \frac{(2n+1)^3}{x^n}$$

$$= \frac{\cancel{x^n}\cdot x}{(2n+3)^3} \cdot \frac{(2n+1)^3}{\cancel{x^n}}$$

$$\left|\frac{U_{n+1}}{U_n}\right| = \frac{x(2n+1)^3}{(2n+3)^3}$$

Divide both numerate and denominator by n

$$= \frac{x(2+1/n)^3}{(2+3/n)^3}$$

$$\lim_{x\to\infty}\left|\frac{U_{n+1}}{U_n}\right| = x,$$

For absolute convergence $\lim_{x\to\infty}\left|\frac{U_{n+1}}{U_n}\right| < 1$

For convergence $|x| < 1$

$$= \frac{x(2)^3}{2^3} \Rightarrow \dots -1 < x < 1$$

Case 1 $|x| < 1$, Put $x = 1$

$$U_n = \frac{1}{(2n+1)^3} < \frac{1}{n^3} \text{ for } P = 5$$

$\therefore x = 1$ (for absolute convergence)

Case 1: for $|x| < 1$, Put $x = -1$

$$U_n = \frac{(-1)^n}{(2n+1)^n} < \frac{1}{n^2}$$

$$= \frac{1}{(2n+1)^3} > \frac{1}{n^3} \text{ for } P = 3$$

$x = -1$ (for absolute convergence)

$\therefore$ The range of values for x for which the series U_n is absolutely convergent is:

$$-1 \leq x \leq 1$$

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