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Course :- ENG 281
Department :- Mech Eng.
Matric No :- 16/ENG06/001

1) $x = \cos t + t\sin t$.

$y = \sin t - t\cos t$

(a) An expression for the radius of curvature (R) in terms of t.

Answer.

$x = \cos t + t\sin t$.

$\frac{dx}{dt} = -\sin t + t\cos t + \sin t = t\cos t$.

$y = \sin t - t\cos t$.

$\frac{dy}{dt} = \cos t + t\sin t - \cos t = t\sin t$

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$= \frac{t\sin t}{t\cos t}$.

$\frac{d^2y}{dx^2} = u = \sin t \qquad v = \cos t$.

$du/dt = \cos t \qquad dv/dt = -\sin t$.

$\frac{v\,du/dt - u\,dv/dt}{v^2}$.

$\frac{\cos t(\cos t) - (\sin t)(-\sin t)}{(\cos t)^2} \times \frac{1}{t\cos t}$

$\frac{\cos^2 t + \sin^2 t}{\cos^2 t} \times \frac{1}{t\cos t}$

$\frac{1}{\cos^2 t} * \frac{1}{t\cos t} = \frac{1}{t\cos^3 t}$

$$\frac{d^2y}{dx^2} = \frac{1}{t\cos^2 t}$$

$$R = \frac{(1 + (dy/dx)^2)^{3/2}}{d^2y/dx^2}$$

$$R = \frac{\left(1 + \frac{\sin^2 t}{\cos^2 t}\right)^{3/2}}{\frac{1}{t\cos^3 t}}$$

$$R = \frac{\left(\frac{\cos^2 t + \sin^2 t}{\cos^2 t}\right)^{3/2}}{\frac{1}{t\cos^3 t}}$$

$$R = \frac{\left(\frac{\cos^2 t + \sin^2 t}{\cos^2 t}\right)^{3/2}}{\frac{1}{t\cos^3 t}}$$

$$R = \frac{(1/\cos^2 t)^{3/2}}{1/t\cos^3 t}$$

$$R = \frac{1/\cos^3 t}{1/t\cos^3 t}$$

$$R = \frac{1}{\cos^3 t} \times t\cos^3 t$$

$$R = t,$$

b) Expressions for the coordinates $A(h, k)$ or the centre of curvature in terms of t.

Answer.

$$h = x_1 - R\sin\theta$$

$$k = y_1 + R\cos\theta$$

$$R = t$$

$$\theta = \tan^{-1}(dy/dx)$$

$\frac{dy}{dt} = \frac{\sin t}{\cos t} = \tan t$

$\therefore \theta = \frac{1}{\tan} \times \tan t$

$\theta = t$

$h = x_1 - t\sin t$

$k = y_1 + t\cos t$

But $x = \cos t + t\sin t$

$y = \sin t - t\cos t$

$h = \cos t + t\sin t - t\sin t.$

$h = \cos t$

$k = \sin t - t\cos t + t\cos t$

$k = \sin t.$

$(h, k) = (\cos t, \sin t).$