AMADI ACHINIKE S. J
PET. ENGR
15/MTTS01/021

1) $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$

Soln

$y'' - y' - 2y = 8$

$y = e^{kx}$, $y' = ke^{kx}$ $y'' = k^2e^{kx}$

$k^2e^{kx} - ke^{kx} - 2ke^{kx} = 8$

$(k^2 - k - 2)e^{kx} = 8$

Since $y = e^{kx}$

$\therefore k^2 - k - 2 = 0$

$k^2 - 2k - k - 2 = 0$

$(k-2)(k+1) = 0$

$k_1 = 2$ $k_2 = -1$

$y_h = C_1e^{2x} + C_2e^{-x}$

Considering the particular, y_p

$y_p = A$

$y'_p = 0$

$y''_p = 0$

Thus $-2A = 8$

$A = -4$

The General Solution

$y = y_h + y_p$

$y = C_1e^{2x} + C_2e^{-x} - 4$

3) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}$

Soln

Assume Homogenity; $y'' + 2y' + y = 0$

$y = e^{kx}$, $y' = ke^{kx}$ $y'' = k^2e^{kx}$

$k^2e^{kx} + 2ke^{kx} + e^{kx} = 0$

$(k^2 + 2k + 1)e^{kx} = 0$

But $y = e^{kx}$

$\therefore (k^2 + 2k + 1) = 0$

$k^2 + k + k + 1 = 0$

$(k+1)(k+1) = 0$

$k_1 = k_2 = -1$ hence

the repeated roots.

Thus,

$y_h = C_1y_1 + xC_2y_2$

$y_h = C_1e^{-x} + xC_2e^{-x}$

The NON-homogenous part

Let, $y_p = Ae^{-2x}$

$y'_p = -2Ae^{-2x}$

$y''_p = 4Ae^{-2x}$

$\therefore 4Ae^{-2x} + 2(-2Ae^{-2x}) + Ae^{-2x} = e^{-2x}$

$4Ae^{-2x} - 4Ae^{-2x} + Ae^{-2x} = e^{-2x}$

$Ae^{-2x} = e^{-2x}$

$A = 1$

$y_p = e^{-2x}$

$y = y_h + y_p$

$y = C_1e^{-x} + xC_2e^{-x} + e^{-2x}$

$y = e^{-x}(C_1 + xC_2) + e^{-2x}$

Scanned by CamScanner

2) $\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

Soln

$y'' - 4y = 10e^{3x}$

$y'' = k^2e^{kx}$, $y' = ke^{kx}$ $y = e^{kx}$

ASSUME HOMOGENEITY

$k^2e^{kx} - 4e^{kx} = 0$

$e^{kx}(k^2-4) = 0$

But $y = e^{kx}$

$k^2 - 4 = 0$

$k^2 = 4$

$k = \pm\sqrt{4}$

$k_1 = 2$, $k_2 = -2$

$y_h = C_1y_1 + C_2y_2$

$y_h = C_1e^{2x} + C_2e^{-2x}$

Considering the particular soln

Let

$y_p = Ae^{3x}$

$y_p' = 3Ae^{3x}$

$y_p'' = 9Ae^{3x}$

Substituting in question

$9Ae^{3x} - 4Ae^{3x} = 10e^{3x}$

$5Ae^{3x} = 10e^{3x}$

$5A = 10$

$A = 2$

$y_p = 2e^{3x}$

The General solution

$y = y_h + y_p$

$y = C_1e^{2x} + C_2e^{-2x} + 2e^{3x}$

4) $\frac{d^2y}{dx^2} + 25y = 5x^2 + x$

Soln

$y'' + 25y = 0$ (Assuming Homogeneity)

$k^2e^{kx} + 25e^{kx} = 0$ (since $y = e^{kx}$)

$k^2 + 25 = 0$

$k^2 = -25$

$k = \pm\sqrt{-25}$

$k_1 = 5i$ $k_2 = -5i$

Since we have complex roots

$y_h = A\cos 5x + B\sin 5x$

The particular solution

Let $y_p = Ax^2 + Bx + C$

$y_p' = 2Ax + B$

$y_p'' = 2A$

Subst in question,

$2A + 25(Ax^2 + Bx + C) = 5x^2 + x$

$2A + 25Ax^2 + 25Bx + 25C = 5x^2 + x$

Comparing Coefficients & collecting terms

$25A = 5$ ∴ $A = 1/5$

$25B = 1$; $B = 1/25$

$2A + 25C = 0$ . . . *

Substituting A in *

$2(1/5) + 25C = 0$ ∴ $C = \frac{-2}{125}$

$y_p = \frac{1}{5}x^2 + \frac{1}{25}x - \frac{2}{125}$

$y = y_h + y_p$

$y = A\cos 5x + B\sin 5x + \frac{1}{5}x^2 + \frac{1}{25}x - \frac{2}{125}$

5)

Scanned by CamScanner

AMADI ACHINIKE S. J 15/MATH501/021

5) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$

Soln

Assuming Homogenity

$y'' - 2y' + y = 0$

$k^2e^{kx} - 2ke^{kx} + e^{kx} = 0$

Since $y = e^{kx}$ $y' = ke^{kx}$ $y'' = k^2e^{kx}$)

$(k^2 - 2k + 1) = 0$

$k^2 - 2k + 1 = 0$

$(k-1)(k-1) = 0$

$k_1 = k_2 = 1$ twice

Repeated roots.

$y_h = \cancel{A\sin x + B\cos x}$

$y_h = (C_1 + xC_2)e^x$

Considering the NON-HOMOGENEOUS PART

Let $y_p = A\sin x + B\cos x$

$y'_p = A\cos x - B\sin x$

$y''_p = -A\sin x - B\cos x$

$-A\sin x - B\cos x - 2(A\cos x - B\sin x) + A\sin x + B\cos x = 4\sin x$

$\cancel{-A\sin x} - B\cos x - 2A\cos x - 2B\sin x \cancel{+A\sin x} + B\cos x = 4\sin x$

$2B\sin x - 2A\cos x = 4\sin x + 0\cos x$

Comparing Coefficients

$2B = 4$; $B = 2$

$-2A = 0$ $A = 0$

$A = 0, B = 2$

$y_p = 2\cos x$

$y = y_h + y_p$

$y = e^x(C_1 + C_2x) + 2\cos x$

$y = e^x(C_1 + xC_2) + 2\cos x$

6) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$

$x = 0, y = 1$ and $y' = -2$.

Soln

$y'' + 4y' + 5y = 0$ (Assume Homogenity

$k^2e^{kx} + 4ke^{kx} + 5e^{kx} = 0$

$(k^2 + 4k + 5) = 0$ Since $y = e^{kx}$

Solving using Completing the square

$k^2 + 4k + 4 = -5 + 4$

$(k+2)^2 = -1$

$k + 2 = \pm\sqrt{-1}$

$k_1 = -2 + i$, $k_2 = -2 - i$

$y_h = e^{-2x}(C_1\cos x + C_2\sin x)$

Let $y_p = Ae^{-2x}$

$y'_p = 2Ae^{-2x}$

$y''_p = 4Ae^{-2x}$

$4Ae^{-2x} + 4(-2Ae^{-2x}) + 5Ae^{-2x} = 2e^{-2x}$

$4Ae^{-2x} + (-8Ae^{-2x}) + 5Ae^{-2x} = 2e^{-2x}$

$Ae^{-2x} = 2e^{-2x}$

$A = 2$

$\therefore y = e^{-2x}(C_1\cos x + C_2\sin x) + 2e^{-2x}$

$y' = 2e^{-2x}(-C_1\sin x + C_2\cos x) + 2e^{-2x}$

$(C_1\cos x + C_2\sin x) - 4e^{-2x}$

from $u\frac{dv}{dx} + v\frac{du}{dx}$; Differentiation

$x = 0, y = 1, y' = -2$

$\Rightarrow y = 1 = e^{2(0)}(C_1\cos(0) + C_2\sin(0) + 2e^{-2(0)}$

$1 = C_1 + 2$

$C_1 = -1$

$y' = -2 = e^{-2(0)}(-C_1\sin(0) + C_2\cos(0) + (-2e^{2(0)})$

$(C_1\cos(0) + C_2\sin(0)) - 4e^{-2(0)}$

$-2 = C_2 - 2C_1 - 4$

$C_2 - 2C_1 = 2$

$C_2 - 2(-1) = 2$

AMADI ACHINIKE S.J

U/MHSO/021 PET-ENG

$C_2 + 2 = 2$

$C_2 = 0$

$y = e^{-2x}(-\cos x + 0) + 2e^{-2x}$

$y = e^{-2x}(-\cos x) + 2e^{-2x}$

$y = e^{-2x}(2 - \cos x)$

7) $3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 2x - 3$

Soln

$3y'' - 2y' - y = 0$ (Assume Homogenity)

$3k^2e^{kx} - 2ke^{kx} - e^{kx} = 0$

$(3k^2 - 2k - 1)e^{kx} = 0$

Since $y = e^{kx}$

$(3k^2 - 2k - 1) = 0$

$(3k+1)(k-1) = 0$

$k_1 = 1, k_2 = 1/3$

$y_h = C_1e^{x} + C_2e^{-x/3}$

The NON-HOMOGENOUS PART

$y_p = Ax - B$

$y'_p = A$

$y''_p = 0$

$3(0) - 2(A) - (Ax - B) = 2x - 3$

$-2A - Ax + B = 2x - 3$

$-A = 2 \therefore A = -2$

$-2A + B = -3$

$B = -7$

Thus $y_p = -2x - (-7)$

$y_p = -2x + 7$

$\therefore$ The Gen solution is given as

$y = y_h + y_p$

$y = C_1e^{x} + C_2e^{-x/3} - 2x + 7$

8) $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$

Soln

$y'' - 6y' + 8y = 0$ (Assume Homogenity)

Recall that $y = e^{kx}$, $y' = ke^{kx}$, $y'' = k^2e^{kx}$

$k^2e^{kx} - 6ke^{kx} + 8e^{kx} = 0$

$e^{kx}(k^2 - 6k + 8) = 0$

$y = e^{kx} \neq 0$

Thus $k^2 - 6k + 8 = 0$

$k^2 - 4k - 2k + 8 = 0$

$(k-2)(k-4) = 0$

$k_1 = 2, k_2 = 4$

$y_h = C_1e^{2x} + C_2e^{4x}$

Consider the particular/Non-Homogenous part

$y_p = Axe^{4x}$

Rewriting

$y_p = xAe^{4x}$

$y'_p = x \cdot 4Ae^{4x} + 1Ae^{4x}$

$y'_p = 4Axe^{4x} + Ae^{4x}$

$y''_p = 16Axe^{4x} + 4Ae^{4x} + 4Ae^{4x}$

$y''_p = 16Axe^{4x} + 8Ae^{4x}$

$16Axe^{4x} + 8Ae^{4x} - 6(Axe^{4x} + Ae^{4x}) + 8(Axe^{4x}) = 8e^{4x}$

$\cancel{16Axe^{4x}} + \cancel{8Ae^{4x}} - 24Axe^{4x} - 6Ae^{4x} + 8Axe^{4x} = 8e^{4x}$

$2Ae^{4x} = 8e^{4x}$

$2A = 8$

$A = 4$

$y_p = 4xe^{4x}$

$\therefore$ The Gen Solution

$y = y_h + y_p$

$y = C_1e^{3x} + C_2e^{2x} + 4xe^{4x}$

Scanned by CamScanner