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ASSIGNMENT

1. The parametric equations of a curve are given in Equations (1) and (2).

$$x = \cos t + t \sin t \quad -- (1)$$

$$y = \sin t - t \cos t \quad -- (2)$$

In terms of t, determine

(i) Expression for radius of curvature (R) and,

(ii) Expression for the coordinates (h, k) of the centre of curvature

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(i) Given R radius of curvature, $R = \dfrac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} \; ; \; \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

$$\frac{dy}{dt} = \cos t - (-t \sin t + \cos t) \quad \text{[from eqn 2]}$$

$$\frac{dy}{dt} = \cos t + t \sin t - \cos t$$

$$\frac{dy}{dx} = t \sin t$$

from eqn (1) $x = \cos t + t \sin t$

$$\frac{dx}{dt} = -\sin t + (t \cos t + \sin t)$$

$$\frac{dx}{dt} = t \cos t \quad \text{but } \frac{dt}{dx} = \frac{1}{t \cos t}$$

$$\frac{dy}{dx} = \frac{dy}{dx} \cdot \frac{dt}{dx} = t \sin t \times \frac{1}{t \cos t} = \frac{\sin t}{\cos t}$$

Recall $\frac{\sin \theta}{\cos \theta} = \tan t$.

$$\frac{dy}{dx} = \tan t$$

for $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dt}\left(\frac{dy}{dx}\right)\frac{dt}{dx}$

$\therefore \frac{d}{dt}(\tan t) \times \frac{1}{t\cos t} \Rightarrow \sec^2 t \times \frac{1}{t\cos t}$

$$\frac{d^2y}{dx^2} = \frac{\sec^2 t}{t\cos t}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{1}{\cos^2 t} \div t\cos t = \frac{1}{\cos^2 t} \times \frac{1}{t\cos t} = \frac{1}{t\cos^3 t}$$

Then $R = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{d^2y/dx^2} = \frac{(1 + \tan^2 t)^{3/2}}{1/t\cos^3 t}$

Recall $1 + \tan^2\theta = \sec^2\theta$

$$\frac{(\sec^2 t)^{3/2}}{1/t\cos^3 t} = \frac{(\sqrt{\sec^2 t})^3}{1/t\cos^3 t}$$

$$R = \sec^3 t \times t\cos^3\theta = \frac{1}{\cos^3 t} \times t\cos^3 t$$

$$R = t$$

but $\tan\theta = \frac{dy}{dx}$ $\therefore \tan\theta = \tan t$ $\therefore \theta = t$ (angle of slope)

ii) a.) $h = x_1 - R\sin t$

$x_1 = \cos t + t\sin t$, $R = t$

$h = \cos t + t\sin t - t\sin t$

$h = \cos t$

b.) $k = y_1 + R\cos t$

$y_1 = \sin t - t\cos t$, $R = t$

$k = \sin t - t\cos t + t\cos t$

$k = \sin t$