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16/ENG07/034
Petroleum Engineering

1) $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$ — ①

When $f(x) = 0$

$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$

$m^2 - m - 2 = 0$

$m^2 - 2m + m - 2 = 0$

$(m^2 - 2m) + (m - 2) = 0$

$m(m-2) + 1(m-2) = 0$

$(m+1)(m-2)$

$m = -1$ or $m = 2$

$CF = y = Ae^{2x} + Be^{-x}$

To get PI

let $y = c$

$\frac{dy}{dx} = 0$

$\frac{d^2y}{dx^2} = 0$

— ① becomes

$-2c = 8$

$c = \frac{8}{-2}$

$c = -4$

$PI = y = -4$

$GS = CF + PI$

$y = Ae^{2x} + Be^{-x} - 4$

2) $\frac{d^2y}{dx^2} - 4y = 10e^{3x}$ — ①

$f(x) = 0$

$\frac{d^2y}{dx^2} - 4y = 0$

$m^2 - 4 = 0$

$m = \pm\sqrt{4}$

$m = +2$ or -2

$y = Ae^{2x} + Be^{-2x}$ — CF

To get P.I

let $y = Ce^{3x}$, $dy/dx = 3Ce^{3x}$,

$\frac{d^2y}{dx^2} = 9Ce^{3x}$

① becomes

$9Ce^{3x} - 4(Ce^{3x}) = 10e^{3x}$

$9Ce^{3x} - 4Ce^{3x} = 10e^{3x}$

$Ce^{3x}(9-4) = 10e^{3x}$

$5C\cancel{e^{3x}} = 10\cancel{e^{3x}}$

$c = 2$

$PI = y = 2e^{3x}$

$G.S = CF + PI$

$y = Ae^{2x} + Be^{-2x} + 2e^{3x}$

3 $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}$

when $f(x) = 0$

$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$

$m^2 + 2m + 1 = 0$

$m^2 + m + m + 1 = 0$

$(m^2 + m)(m + 1) = 0$

$m(m+1) + 1(m+1) = 0$

$(m+1)(m+1) = 0$

$m = -1$ twice

$y = e^{-x}(A + Bx)$ - CF

to get PI

let $y = Ce^{-2x}$

$dy/dx = -2Ce^{-2x}$

$\frac{d^2y}{dx^2} = 4Ce^{-2x}$

① becomes

$4Ce^{-2x} + 2(-2Ce^{-2x}) + Ce^{-2x} = e^{-2x}$

$4Ce^{-2x} - 4Ce^{-2x} + Ce^{-2x} = e^{-2x}$

$Ce^{-2x}(4 - 4 + 1) = e^{-2x}$

$\cancel{Ce^{-2x}} = \cancel{e^{-2x}}$

$C = 1$

$PI = y = e^{-2x}$

GS = CF + PI

$y = e^{-x}(A + Bx) + e^{-2x}$

4) $\frac{d^2y}{dx^2} + 25y = 5x^2 + x$ - ①

let $f(x) = 0$

$\frac{d^2y}{dx^2} + 25y = 0$

$m^2 + 25 = 0$

$m^2 = -25$

$m = \pm\sqrt{-25}$

$m = \pm i5$

$y = C\cos 5x + D\sin 5x$ - CF

to get PI

$y = Cx^2 + Dx + E$

$\frac{dy}{dx} = 2Cx + D$

$\frac{d^2y}{dx^2} = 2C$

① becomes

$2C + 25(Cx^2 + Dx + E) = 5x^2 + x$

$2C + 25Cx^2 + 25Dx + 25E = 5x^2 + x$

$25C = 5$

$C = 1/5$

$25D = 1$

$D = 1/25$

$2C + 25E = 0$

$2(1/5) + 25E = 0$

$2/5 + 25E = 0$

$25E = -2/5$

$E = \frac{-2}{125}$

$PI = y = \frac{x^2}{5} + \frac{x}{25} - \frac{2}{125}$

$GS = C\cos 5x + D\sin 5x + \frac{x^2}{5} + \frac{x}{25} - 2/125$

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5 $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$

$f(x) = 0$

$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$

$m^2 - 2m + y = 0$

$m^2 - m - m + 1 = 0$

$(m^2 - m)(-m + 1) = 0$

$m(m-1) - 1(m-1) = 0$

$(m-1)(m-1)$

$m_1 = 1$ twice

$y = e^x(A + Bx)$ - CF

to get PI

let $y = C\cos x + D\sin x$

$dy/dx = -C\sin x + D\cos x$

$\frac{d^2y}{dx^2} = -C\cos x - D\sin x$

- ① become

$-C\cos x - D\sin x - 2C(-C\sin x$

$+ D\cos x) + C\cos x$

$+ D\sin x = 4\sin x$

$-C\cos x - D\sin x + 2C\sin x$

$- 2D\cos x + C\cos x + D$

$\sin x = 4\sin x$

$\cos x(-C - 2D + C) + \sin x$

$(-D + 2C + D) = 4\sin x$

$\cos x(-2D) + \sin x(2C) =$

$4\sin x$

$\cos x(-2D) + \sin x(2C) =$

$4\sin x$

$-2D = 0$

$D = 0$

$2C = 4$

$C = 2$

$PI = y = 2\cos x + 0\sin x$

$y = 2\cos x$

G.S = P.I + CF

$y = e^x(A + Bx) + 2\cos x$

6) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$

$x = 0, y = 1$ and $dy/dx = -2$

$f(x) = 0$

$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 0$

$m^2 + 4m + 5 = 0$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\frac{-4 \pm \sqrt{16 - 4 \times 5}}{2}$

$\frac{-4 \pm \sqrt{16 - 20}}{2}$

$\frac{-4 \pm \sqrt{-4}}{2}$

$\frac{-4 \pm j\sqrt{4}}{2}$

$-2 \pm j$, $y = e^{-2x}(C\cos x + D\sin x$

To get PE

The gen. form of the RHS $= Ce^{-2x}$

but the term e^{-2x} is already

part in the C.F so, we

assume

$y = Cxe^{-2x}$

$$\frac{dy}{dx} = Cx(-2e^{-2x}) + e^{-2x}(C)$$
$$= -2Cxe^{-2x} + Ce^{-2x}$$

$$\frac{d^2y}{dx^2} = -2Cx(-2e^{-2x}) + e^{-2x}(-2C) + -2Ce^{-2x}$$

$$= 4Cxe^{-2x} - 2Ce^{-2x} - 2Ce^{-2x}$$
$$= 4Cxe^{-2x} - 4Ce^{-2x}$$

① becomes

$$4Cxe^{-2x} - 4Ce^{-2x} + 4(-2Cxe^{-2x} + Ce^{-2x}) + 5(Cxe^{-2x}) = 2e^{-2x}$$

$$4Cxe^{-2x} - 4Ce^{-2x} - 8Cxe^{-2x} + 4Ce^{-2x} + 5Cxe^{-2x} = 2e^{-2x}$$

$$4Cxe^{-2x} - 8Cxe^{-2x} + 5Cxe^{-2x} \cancel{-4Ce^{-2x}} + \cancel{4Ce^{-2x}} = 2e^{-2x}$$

$$\cancel{e^{-2x}}(4Cx - 8Cx + 5Cx) = 2\cancel{e^{-2x}}$$
$$Cx = 2$$

P.I, $y = 2e^{-2x}$

$$y = e^{-2x}(C\cos x + D\sin x) + 2e^{-2x} \quad \text{— Gs}$$

at $x = 0$ and $y = 1$

$$1 = e^{-2(0)}(C\cos(0) + D\sin(0)) + 2e^{-2(0)}$$
$$1 = 1(C + 0) + 2$$
$$1 = C + 2$$
$$C = -1$$

$$\frac{dy}{dx} = e^{-2x}(-C\sin x + D\cos x) + (C\cos x + D\sin x) - 2e^{-2x} - 4e^{-2x}$$

at $x = 0$, $y = 1$ and $\frac{dy}{dx} = -2$

$$-2 = D - 2C - 4$$
$$-2 + 4 = D - 2C$$
$$2 = D + 2$$
$$D = 0$$

$$y = e^{-2x}(-\cos x + 0) + 2e^{-2x}$$
$$y = e^{-2x}(-\cos x) + 2e^{-2x}$$
$$y = e^{-2x}(2 - \cos x)$$

7) $3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 0$

$$3m^2 - 2m - 1 = 0$$
$$3m^2 - 3m + m - 1 = 0$$
$$(3m^2 - 3m) + (m - 1) = 0$$
$$3m(m-1) + 1(m-1) = 0$$
$$(3m+1)(m-1) = 0$$
$$m = 1, \quad m = -1/3$$
$$y = Ae^{x} + Be^{-\frac{1}{3}x}$$

To find PI

let $y = Cx + D$

$$\frac{dy}{dx} = C$$

$$\frac{d^2y}{dx^2} = 0$$

-① becomes

$$0 - 2C - Cx - D = 2x - 3$$
$$-Cx - 2C - D = 2x - 3$$
$$-Cx = 2x$$
$$-C = 2$$

$-2c - b = -3$

$+4 - b = -3$

$-b = -7, b = 7, PI = -2x+7$

$= -2x + 7$

$y = Ae^{x} + Be^{-1/3x} - 2x + 7$

8 $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$ — (1)

$f(x) = 0$

$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 0$

$m^2 - 6m + 8 = 0$

$m^2 - 4m - 2m + 8 = 0$

$(m^2 - 4m)(-2m + 8) = 0$

$m(m-4) - 2(m-4) = 0$

$(m-4)(m-2)$

$m = 4$ or 2

$y = Ae^{4x} + Be^{2x}$ — CF

To get PI

The general form of the RHS is Ce^{4x} but this term e^{4x} is already contained in the CF, so assume,

$y = Cxe^{4x}$

$\frac{dy}{dx} = Cx(4e^{4x}) + Ce^{4x}$

$= 4Cxe^{4x} + Ce^{4x}$

(1) becomes

$16Cxe^{4x} + 4Ce^{4x} + 4Ce^{4x} - 6C(4e^{4x} + Ce^{4x}) + 8C(Cxe^{4x}) =$

$\frac{d^2y}{dx^2} = \cancel{16Cxe^{4x}} + \cancel{4Ce^{4x}} + 4Ce^{4x}$

$16Cxe^{4x} + 4Ce^{4x} + 4Ce^{4x} - 24Cxe^{4x} - 6Ce^{4x} + 8Cxe^{4x} = 8e^{4x}$

$16Cxe^{4x} - 24Cxe^{4x} + 8Cxe^{4x} + 4Ce^{4x} + 4Ce^{4x} - 6Ce^{4x} = 8e^{4x}$

$e^{4x}(0 + 2C) = 8e^{4x}$

$2C = 8$

$C = 4$

$y = 4xe^{4x}$

G.S

C.F + PI

$Ae^{4x} + Be^{2x} + 4xe^{4x}$ //