15/ENG06/054
OMAJUGHO SPENCER
MECHANICAL ENGINEERING
ENG 381
ENGINEERING MATHS III
ASSIGNMENT 1

Question 1

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$$

$$M^2 - M - 2 = 0$$

$$M_1 = 2, \quad M_2 = -1$$

$$y = Ae^{2x} + Be^{-x}$$

PI : $y = C$

$$\frac{dy}{dx} = 0$$

$$\frac{d^2y}{dx^2} = 0$$

$$0 - 0 - 2C = 8$$

$$-2C = 8$$

$$C = \frac{-8}{2}$$

$$C = -4$$

PI : $y = -4$

G.S : $y = Ae^{2x} + Be^{-x} + 4$

Question 2

$$\frac{d^2y}{dx^2} - 4y = 10e^{3x}$$

$$M^2 - 4 = 0$$

$$M^2 = 4$$

$$M = \pm\sqrt{4}$$

$$M = \pm 2j$$

$$y = C\cosh 2x + D\sinh 2x$$

PI : $y = Ce^{3x}$

$$\frac{dy}{dx} = 3Ce^{3x}$$

$$\frac{d^2y}{dx^2} = 9Ce^{3x}$$

$$9Ce^{3x} - 4(Ce^{3x}) = 10e^{3x}$$

$$9Ce^{3x} - 4Ce^{3x} = 10e^{3x}$$

$$5Ce^{3x} = 10e^{3x}$$

$$C = \frac{10e^{3x}}{5e^{3x}}$$

$$C = 2$$

PI : $y = 2e^{3x}$

G.S : $C\cosh 2x + D\sinh 2x + 2e^{3x}$

Question 3

$$\frac{d^2y}{dx^2} + \frac{2dy}{dx} + y = e^{-2x}$$

$$m^2 + 2m + 1 = 0$$

$$\frac{-2 \pm \sqrt{(2)^2 - 4 \times 1 \times 1}}{2 \times 1} = \frac{-2}{2} = -1$$

$$m = -1$$

$$y = e^{-x}(A + Bx)$$

PI : $y = Ce^{-2x}$

$$\frac{dy}{dx} = -2Ce^{-2x}$$

$$\frac{dy}{dx} = 4Ce^{-2x}$$

$$4Ce^{-2x} - 2(2Ce^{-2x}) + Ce^{-2x} = e^{-2x}$$

$$4Ce^{-2x} - 4Ce^{-2x} + Ce^{-2x} = e^{-2x}$$

$$Ce^{-2x} = e^{-2x}$$

$$C = \frac{e^{-2x}}{e^{-2x}}$$

$$C = 1$$

$$y = e^{-2x}$$

G.S : $y = e^{-x}(A + Bx)e^{-2x}$

Question 4

$$\frac{d^2y}{dx^2} + 25y = 5x^2 + x$$

$$m^2 + 25 = 0$$

$$m^2 = -25$$

$$m = \pm\sqrt{25}$$

$$m = 5j$$

$$y = C\cos 5x + D\sin 5x$$

PI : $y = Cx^2 + Dx + E$

$$\frac{dy}{dx} = 2Cx + D$$

$$\frac{d^2y}{dx} = 2C$$

$$2C + 25(Cx^2 + Dx + E) = 5x^2 + x$$

$$2C + 25Cx^2 + 25Dx + 25E = 5x^2 + x$$

$25C = 5$ Comparing Coefficient

$$C = \frac{1}{5} \qquad 25D = 1$$

$$D = \frac{1}{25}$$

$$25E + 2C = 0 \qquad 25E + 2\left(\frac{1}{5}\right) = 0$$

$$25E = \frac{-2}{5} \qquad E = \frac{-2}{125}$$

$$y = \frac{1}{5}x^2 + \frac{1}{25}x - \frac{2}{125}$$

G.S : $y = C\cos 5x + D\sin 5x + \frac{1}{5}x^2 + \frac{1}{25}x - \frac{2}{125}$

Question 5

$$\frac{d^2y}{dx} - 2\frac{dy}{dx} + y = 4\sin x$$

$m^2 - 2m + 1$

$m = 1$ (using quadratic Formula)

$y = e^x(A + Bx)$

$\frac{dy}{dx} = -C\sin x + D\cos x$

$\frac{d^2y}{dx^2} = -C\cos x - D\sin x$

$-C\cos x - D\sin x - 2(-C\sin x + D\cos x) + C\cos x + D\sin x$

$= 4\sin x$

$-C\cos x - 2D\cos x + C\cos x - D\sin x + 2C\sin x + D\sin x$

$= 4\sin x$

$-2D\cos x + 2C\sin x = 4\sin x$

Comparing Coefficient

$-2D = 0 \qquad 2C = 4$

$D = 0 \qquad C = 2$

$y = 2\cos x + 0\sin x$

$y = 2\cos x$

G.S: $y = e^x(A + Bx)$

Question 6

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$$

given that $x = 0$, $y = 1$ and $\frac{dy}{dx} = -2$

$m^2 + 4m + 5$

$\frac{-4 \pm \sqrt{(4)^2 - 4\times1\times5}}{2\times1} = \frac{-4 \pm 2}{2} = -2 \pm j$

$e^{-2x}(C\cos 2x + D\sin 2x)$

PI: $y = Ce^{-2x}$

$\frac{dy}{dx} = -2Ce^{-2x}$

$\frac{d^2y}{dx} = 4Ce^{-2x}$

$4Ce^{-2x} + 4(-2Ce^{-2x}) + 5C(e^{-2x}) = 2e^{-2x}$

$4C + 5C - 8C = 2$

$C = 2 \qquad y: 2e^{-2x}$

G.S $y = e^{-2x}(\cos 2x + D\sin 2(0)) + 2e^{-2\cos}$

$1 = C + 2$

$C = -1$

$\frac{dy}{dx} = e^{-2x}(-2(\sin^2 x + 2D\cos^2 x) + C(\cos 2x + D\sin 2x)$

$-4e^{2x}$, at $x = 0 \qquad -2 = D + C$

$-2 = D - 1$

$D = -1$

C.S: $y = e^{-2x}(-C\cos 2x - D\sin 2x)$

Question 7

$$3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 2x - 3$$

$$3m^2 - 2m - 1 = 0$$

$$\frac{2 \pm \sqrt{(-2)^2 - 4 \times 3 \times 1}}{2 \times 3}$$

$$\frac{2 \pm \sqrt{4 - 12}}{6} = \frac{2 \pm \sqrt{12}}{6}$$

$m_1 = 1$ & $m_2 = \frac{-1}{3}$

$$y = Ae^x + Be^{\frac{-1}{3}}$$

PI : $y = Cx + D$

$$\frac{dy}{dx} = C \qquad \frac{d^2y}{dx^2} = 0$$

$$3(0) - 2C - (Cx + D) = 2x - 3$$

$$-2C - Cx + D = 2x - 3$$

Comparing Coefficients

$$C = -2$$

$$-2C - D = -3$$

$$-2(-2) - D = -3$$

$$4 - D = -3$$

$$D = 4 + 3 = 7 \qquad y = -2x + 7$$

G.S : $y = Ae^x + Be^{\frac{-1}{3}} - 2x + 7$

Question 8

$$\frac{\Delta^2 y}{\Delta x^2} - 6\frac{\Delta y}{\Delta x} + 8y = 8e^{4x}$$

$$M^2 - 6m + 8 = 0$$

$$M_1 = 4, \quad M_2 = 2$$

$$y = Ae^{4x} + Be^{2x} = e^{4x}$$

PI : $y = Cxe^{4x}$

$$\frac{\Delta y}{\Delta x} = 4(Cxe^{4x} + Ce^{4x}$$

$$\frac{\Delta^2 y}{\Delta x^2} = 16Cxe^{4x} + 8Ce^{4x}$$

$$16Cxe^{4x} + 8Ce^{4x} - 6(4Cxe^{4x} + Ce^{4x}) + 8(Cxe^{4}$$

$$16Cx + 8C - 24Cx + 6C + 8Cx = 8$$

$$2C = 0$$

$$C = 4$$

$$y = 4xe^{4x}$$

G.S : $y = Ae^{4x} + Be^{2x} + 4xe^{4x}$