

16/ENG06/081

KESSINGTON ENAGBARE

MECHANICAL ENGINEERING

ENG 381

ENGINEERING MATHS III

ASSIGNMENT 1

Question 1

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$$

$$M^2 - M - 2 = 0$$

$$M_1 = 2, M_2 = -1$$

$$y = Ae^{2x} + Be^{-x}$$

$$P_1: y = c$$

$$\frac{dy}{dx} = 0$$

$$\frac{d^2y}{dx^2} = 0$$

$$0 - 0 - 2c = 8$$

$$-2c = 8$$

$$c = \frac{-8}{2}$$

$$c = -4$$

$$P_1: y = -4$$

$$G.S: y = Ae^{2x} + Be^{-x} + 4$$

Question 2

$$\frac{d^2y}{dx^2} - 4y = 10e^{3x}$$

$$M^2 - 4 = 0$$

$$M^2 = 4$$

$$M = \pm \sqrt{4}$$

$$M = \pm 2j$$

$$y = C \cosh 2x + D \sinh 2x$$

$$P_1: y = Ce^{3x}$$

$$\frac{dy}{dx} = 3Ce^{3x}$$

$$\frac{d^2y}{dx^2} = 9Ce^{3x}$$

$$9Ce^{3x} - 4(Ce^{3x}) = 10e^{3x}$$

$$9Ce^{3x} - 4Ce^{3x} = 10e^{3x}$$

$$5Ce^{3x} = 10e^{3x}$$

$$C = \frac{10e^{3x}}{5e^{3x}}$$

$$C = 2$$

$$P_1: y = 2e^{3x}$$

$$G.S: C \cosh 2x + D \sinh 2x + 2e^{3x}$$

Question 3

$$\frac{d^2 y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}$$

$$m^2 + 2m + 1 = 0$$

$$\frac{-2 \pm \sqrt{(2)^2 - 4 \times 1 \times 1}}{2 \times 1} = \frac{-2 \pm 0}{2} = -1$$

$$m = -1$$

$$y = e^{-x}(A + Bx)$$

$$P.I : y = Ce^{-2x}$$

$$\frac{dy}{dx} = -2Ce^{-2x}$$

$$\frac{dy}{dx} = 4Ce^{-2x}$$

$$4Ce^{-2x} - 2(2Ce^{-2x}) + Ce^{-2x} = e^{-2x}$$

$$\cancel{4Ce^{-2x}} - \cancel{4Ce^{-2x}} + Ce^{-2x} = e^{-2x}$$

$$Ce^{-2x} = e^{-2x}$$

$$C = \frac{e^{-2x}}{e^{-2x}}$$

$$C = 1$$

$$y = e^{-2x}$$

$$G.S : y = e^{-x}(A + Bx)e^{-2x}$$

Question 4

$$\frac{d^2 y}{dx^2} + 25y = 5x^2 + x$$

$$m^2 + 25 = 0$$

$$m^2 = -25$$

$$m = \pm \sqrt{25}$$

$$m = \pm 5i$$

$$y = C \cosh 5x + D \sinh 5x$$

$$P.I : y = Cx^2 + Dx + E$$

$$\frac{dy}{dx} = 2Cx + D$$

$$\frac{d^2 y}{dx^2} = 2C$$

$$2C + 25(Cx^2 + Dx + E) = 5x^2 + x$$

$$2C + 25Cx^2 + 25Dx^2 + 25E = 5x^2 + x$$

$$25C = 5 \quad \text{Comparing Coefficient}$$

$$C = \frac{1}{5} \quad 25D = 1$$

$$D = \frac{1}{25}$$

$$25E + 2C = 0 \quad 25E + 2\left(\frac{1}{5}\right) = 0$$

$$25E = \frac{-2}{5} \quad E = \frac{-2}{125}$$

$$y = \frac{1}{5}x^2 + \frac{1}{25}x - \frac{2}{125}$$

$$G.S : y = C \cos 5x + D \sin 5x + \frac{1}{5}x^2 + \frac{1}{25}x - \frac{2}{125}$$

Question 5

$$d^2y - 2\frac{dy}{dx} + y = 4\sin x$$

$$M^2 - 2M + 1$$

~~A~~ $M=1$ (using quadratic formula)

$$y = e^x(A+Bx)$$

$$\frac{dy}{dx} = -C\sin x + D\cos x$$

$$\frac{d^2y}{dx^2} = -C\cos x - D\sin x$$

$$-C\cos x - D\sin x - 2(-C\sin x + D\cos x) + C\cos x + D\sin x = 4\sin x$$

$$-C\cos x - 2D\cos x + C\cos x - 2D\sin x + 2C\sin x + D\sin x = 4\sin x$$

$$-2D\cos x + 2C\sin x = 4\sin x$$

Comparing Coefficient

$$-2D = 0 \quad 2C = 4$$

$$D = 0 \quad C = 2$$

$$y = 2\cos x + 0\sin x$$

$$y = 2\cos x$$

G.S: $y = e^x(A+Bx)$

Question 6

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$$

given that $x=0$

$$y=1 \text{ and } \frac{dy}{dx} = -2$$

$$M^2 + 4M + 5$$

$$-4 \pm \frac{\sqrt{(4)^2 - 4 \times 1 \times 5}}{2 \times 1} = \frac{-4 \pm 2}{2} = -2 \pm j$$

$$e^{-2x}(C\cos 2x + D\sin 2x)$$

P.I: $y = Ce^{-2x}$

$$\frac{dy}{dx} = -2Ce^{-2x}$$

$$\frac{d^2y}{dx^2} = 4Ce^{-2x}$$

$$4(Ce^{-2x} + 4(-2Ce^{-2x}) + 5C(e^{-2x})) = 2e^{-2x}$$

$$4C + 5C - 8C = 2$$

$$C = 2 \quad y: 2e^{-2x}$$

G.S $y = e^{-2x}(C\cos 2x + D\sin 2x) + 2e^{-2\cos}$

$$1 = C + 2$$

$$C = -1$$

$$\frac{dy}{dx} = e^{-2x}(-2(\sin^2 x + 2D\cos^2 x) + C(\cos 2x + D\sin 2x))$$

$$-4e^{-2x}, \text{ at } x=0 \quad -2 = D + C$$

$$-2 = D - 1$$

$$D = -1$$

G.S: $y = e^{-2x}(-\cos 2x - D\sin 2x)$

Question 7

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 2x - 3$$

$$m^2 - 2m - 1 = 0$$

$$\frac{2 \pm \sqrt{(-2)^2 - 4 \times 3 \times 1}}{2 \times 3} = \frac{2 \pm \sqrt{4 - 12}}{6}$$

$$= \frac{2 \pm \sqrt{4 - 12}}{6} = \frac{2 \pm \sqrt{-8}}{6}$$

$$m_1 = 1 \quad m_2 = \frac{-1}{3}$$

$$y = Ae^x + Be^{-\frac{1}{3}x}$$

$$PI: y = Cx + D$$

$$\frac{dy}{dx} = 0C \quad \frac{d^2y}{dx^2} = 0$$

$$3(0) - 2C - (Cx + D) = 2x - 3$$

$$-2C - Cx + D = 2x - 3$$

Comparing Coefficients

$$C = -2$$

$$-2C - D = -3$$

$$-2(-2) - D = -3$$

$$4 - D = -3$$

$$D = 4 + 3 = 7$$

$$y = -2x + 7$$

$$G.S: y = Ae^x + Be^{-\frac{1}{3}x} - 2x + 7$$

Question 8

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$$

$$m^2 - 6m + 8 = 0$$

$$m_1 = 4, m_2 = 2$$

$$y = Ae^{4x} + Be^{2x} = e^{4x}$$

$$PI: y = Cxe^{4x}$$

$$\frac{dy}{dx} = 4(Cxe^{4x} + Ce^{4x})$$

$$\frac{d^2y}{dx^2} = 16Cxe^{4x} + 8Ce^{4x}$$

$$16Cxe^{4x} + 8Ce^{4x} - 6(4Cxe^{4x} + Ce^{4x}) + 8(Cxe^{4x}) = 8e^{4x}$$

$$16Cx + 8C - 24Cx - 6C + 8Cx = 8$$

$$-2C = 0$$

$$C = 4$$

$$y = 4xe^{4x}$$

$$G.S: y = Ae^{4x} + Be^{2x} + 4xe^{4x}$$