6/10/17 EXERCISE (Assignment I) from LMS.

AROWOLO OLUWAFEMI ISRAEL
Mechanical Engineering
16/ENG06/015
ENG 281 (Engineering Maths)

Soln.

1. ~~Evaluate~~

(a) $\lim_{x \to \pi/2} \left[\frac{(x^2 - \pi/4)\sin(\cos x)}{x - \pi/2} \right]$

Since the above will give an undefined answer, L'Hospital's rule is used:

$\lim_{x \to \frac{\pi}{2}} \left(\frac{d}{dx} \left(\frac{(x^2 - \pi/4)\sin(\cos x)}{x - \pi/2} \right) \right)$

Numerator $= \underset{u}{(x^2 - \pi/4)} \underset{v}{(\sin(\cos x))}$

$\frac{du}{dx} = 2x$ $\qquad \frac{dv}{dx} = \cos(-\sin x)$

$\therefore \frac{d}{dx}(x^2 - \pi/4)(\sin(\cos x)) = (x^2 - \pi/4)\cos(-\sin x) + 2x(\sin(\cos x))$

Denominator $= x - \pi/2$

$\frac{d}{dx}(x - \pi/2) = 1$

$\therefore \lim_{x \to \pi/2} \left[\frac{(x^2 - \pi/4)\sin(\cos x)}{x - \pi/2} \right] = \lim_{x \to \pi/2} \left[\frac{(x^2 - \pi/4)\cos(-\sin x) + 2x(\sin(\cos x))}{1} \right]$

$= \left(\frac{\pi}{2}\right)^2 - \frac{\pi}{4} \cos(-\sin 90) + 2x(\sin(\cos 90))$

$= \frac{\pi^2}{4} - \frac{\pi}{4} \times 1 + 2x \times 0$

$= \frac{\pi^2}{4} - \frac{\pi}{4}$

$= \frac{\pi^2 - \pi}{4}$

$= \frac{180^2 - 180}{4}$

$= 8055$

(b) $\lim_{x \to \pi/2} \ln\left[\exp \frac{(3x^2+2x-1)}{x+1}\right] = \lim_{x \to \pi/2} \cancel{\ln}\left[\cancel{\exp}\frac{3x^2+2x-1}{x+1}\right]$

$= \lim_{x \to \pi/2} \frac{(3x^2+3x)(-x-1)}{x+1}$

$= \lim_{x \to \pi/2} \frac{(3x-1)\cancel{(x+1)}}{\cancel{x+1}}$

$= \lim_{x \to \pi/2} 3x-1$

$= 3\left(\frac{\pi}{2}\right) - 1$

$= \frac{3}{2} \times \frac{22}{7} - 1$

$= \frac{66}{14} - 1$

$= \frac{26}{7}$

(c) $\lim_{x \to 2+\sqrt{3}} \cos\left(\sin^{-1}\frac{(x-2)}{(x-\sqrt{3})}\right) = \cos\left(\sin^{-1}\frac{(2+\sqrt{3}-2)}{(2+\sqrt{3}-\sqrt{3})}\right)$

$= \cos\left(\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)$

$= \cos 60$

$= \frac{1}{2}$

(d) $\lim_{x \to 4}\left(\frac{x^2-8x+16}{x^2-5x+4}\right)$

Since the above will give an undefined answer, L'Hospital's rule is used:

$\lim_{x \to 4}\left(\frac{x^2-8x+16}{x^2-5x+4}\right) = \lim_{x \to 4}\left(\frac{2x-8}{2x-5}\right)$

$= \frac{8-8}{8-5}$

$= \frac{0}{3}$

$= 0$

②

ⓐ $\frac{2}{2\times3} + \frac{2}{3\times4} + \frac{2}{4\times5} + \frac{2}{5\times6} + \ldots\ldots$

$\therefore U_n = \frac{2}{(n+1)\cdot(n+2)}$

$\therefore U_{n+1} = \frac{2}{(n+1+1)\cdot(n+1+2)}$

$U_{n+1} = \frac{2}{(n+2)\cdot(n+3)}$

Using D'Alembert's ratio

$\lim_{n\to\infty} \frac{U_{n+1}}{U_n} = \lim_{n\to\infty} \frac{\cancel{2}}{(n+2)\cdot(n+3)} \times \frac{(n+1)\cdot(n+2)}{\cancel{2}}$

$= \lim_{n\to\infty} \frac{n+1}{n+3}$

$= \lim_{n\to\infty} \frac{1+1/n}{1+3/n}$

As $n\to\infty$, $1/n$ & $3/n \to 0$

$\lim_{n\to\infty} \frac{U_{n+1}}{U_n} = \frac{1+0}{1+0}$

$\lim_{n\to\infty} \frac{U_{n+1}}{U_n} = 1$

Since $\lim_{n\to\infty} \frac{U_{n+1}}{U_n} = 1$, meaning it may converge or diverge,

we further test,

$\lim_{n\to\infty} U_n = \frac{2}{(n+1)\cdot(n+2)}$

$= \frac{2}{n(n+2)+1(n+2)}$

$= \frac{2}{n^2+2n+n+2}$

$= \frac{2}{n^2+3n+2}$

$= \frac{2/n^2}{1+3/n+2/n^2}$

As $n\to\infty$, $2/n^2$ & $3/n \to 0$

$\lim_{n\to\infty} U_n = \frac{0}{1+0+0}$

$\therefore \lim_{n\to\infty} U_n = 0$, which indicates the series may be convergent

We test further using the comparison test:
Compare, from the third values, the series $\frac{2}{(n+1)\cdot(n+2)}$ with

the series $\frac{2}{2^n}$

$\therefore$ for $\frac{2}{(n+1)\cdot(n+2)} = \frac{2}{2\times3} + \frac{2}{3\times4} + \frac{2}{4\times5} + \frac{2}{5\times6} + \ldots\ldots$

and for $\frac{2}{2^n} = \frac{2}{2} + \frac{2}{2^2} + \frac{2}{2^3} + \frac{2}{2^4} + \ldots\ldots$

$\therefore$ Since $\frac{2}{20} < \frac{2}{8}$ and $\frac{2}{30} < \frac{2}{16}$, then we can say the series therefore, is <u>convergent.</u>

(6) $\frac{2}{1^2} + \frac{2}{2^2} + \frac{2}{3^2} + \frac{2}{4^2} + \ldots\ldots$

$\therefore U_n = \frac{2}{n^2}$

$U_{n+1} = \frac{2}{(n+1)^2}$

$U_{n+1} = \frac{2}{n(n+1)+1(n+1)}$

$U_{n+1} = \frac{2}{n^2+n+n+1}$

$U_{n+1} = \frac{2}{n^2+2n+1}$

$\therefore \lim_{n\to\infty} \frac{U_{n+1}}{U_n} = \frac{\cancel{2}}{n^2+2n+1} \times \frac{n^2}{\cancel{2}}$

$= \lim_{n\to\infty} \frac{n^2}{n^2+2n+1}$

$= \lim_{n\to\infty} \frac{1}{1+2/n+1/n^2}$

$\therefore$ As $n \to \infty$, $2/n$ & $1/n^2 \to 0$

$\therefore \lim_{n\to\infty} \frac{U_{n+1}}{U_n} = \frac{1}{1+0+0}$

$$\lim_{n\to\infty} \frac{U_{n+1}}{U_n} = 1$$

$\therefore$ Since $\lim_{n\to\infty} \frac{U_{n+1}}{U_n} = 1$, this means it may converge or diverge,

so we further test,

Using Comparison test, we compare from the third values, the series $\frac{2}{n^2}$ with the series $\frac{1}{n^2}$

for $\frac{2}{n^2} = \frac{2}{1^2} + \frac{2}{2^2} + \frac{2}{3^2} + \frac{2}{4^2} + \ldots$

~~$\frac{2}{2^n} = \frac{2}{2^1} + \frac{2}{2^2} + \frac{2}{2^3} + \frac{2}{2^4} + \ldots$~~

and for $\frac{1}{n^2} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \ldots$

$\therefore$ Since $\frac{2}{3^2} > \frac{1}{3^2}$

and $\frac{2}{4^2} > \frac{1}{4^2}$

$\therefore$ Since $\frac{2}{n^2}$ series $> \frac{1}{n^2}$ series, therefore the $2/n^2$ series is said to be divergent.

ⓒ $U_n = \frac{1+2n^2}{1+n^2}$

$\therefore \lim_{n\to\infty} U_n = \lim_{n\to\infty} \frac{1/n^2 + 2}{1/n^2 + 1}$

As $n \to \infty$, $1/n^2 \to 0$

$\therefore \lim_{n\to\infty} U_n = \frac{0+2}{0+1}$

$\lim_{n\to\infty} U_n = \frac{2}{1}$

$\therefore$ Since $\lim_{n\to\infty} U_n \neq 0$, the series is divergent.

(3) $\frac{x}{27} + \frac{x^2}{125} + \dots\dots + \frac{x^n}{(2n+1)^3}$

$U_n = \frac{x^n}{(2n+1)^3}$

$\therefore U_{n+1} = \frac{x^{n+1}}{(2(n+1)+1)^3}$

$U_{n+1} = \frac{x^{n+1}}{(2n+3)^3}$

Using D'Alembert's ratio,

$$\lim_{n\to\infty} \frac{U_{n+1}}{U_n} = \lim_{n\to\infty} \frac{x^{n+1}}{(2n+3)^3} \times \frac{(2n+1)^3}{x^n}$$

$$= \lim_{n\to\infty} \frac{(x^{n+1-n})(2n+1)^3}{(2n+3)^3}$$

$$= \lim_{n\to\infty} \frac{x(8n^3+12n^2+8n+2)}{(8n^3+36n^2+54n+27)}$$

$$= \lim_{n\to\infty} \frac{8xn^3+12xn^2+8xn+2x}{8n^3+36n^2+54n+27}$$

$$= \lim_{n\to\infty} \frac{8x + \frac{12x}{n} + 8x/n^2 + 2x/n^3}{8 + 36/n + 54/n^2 + 27/n^3}$$

$\therefore$ as $n \to \infty$, $\frac{12}{n}, \frac{8}{n^2}, \frac{2}{n^3}, \frac{36}{n}, 54/n^2$ & $\frac{27}{n^3} \to 0$

$$\therefore \lim_{n\to\infty} \frac{U_{n+1}}{U_n} = \frac{8x+0+0+0}{8+0+0+0}$$

$$= x$$

$\therefore$ Since it is convergent

$$\lim_{n\to\infty} \frac{U_{n+1}}{U_n} < 1$$

$$x < 1$$

(4) $\lim_{x\to 0}\left[\frac{\sin x - \cos x}{x^3}\right]$ ~~Using L'Hospital's Rule~~ $= \lim_{x\to 0}\left[\frac{\cos x + \sin x}{3x^2}\right]$

~~Using L'Hospital's Rule~~

$$\lim_{x\to 0}\left[\frac{\sin x - \cos x}{x^3}\right] = \lim_{x\to 0}\left[\frac{-\sin x + \cos x}{6x}\right]$$

$$\lim_{x\to 0}\left[\frac{\sin x - \cos x}{x^3}\right] = \lim_{x\to 0}\left[\frac{-\cos x - \sin x}{6}\right]$$

$$\lim_{x \to 0} \left[\frac{\sin x - \cos x}{x^3}\right] = \frac{-\cos 0 - \sin 0}{6}$$

$$\lim_{x \to 0} \left[\frac{\sin x - \cos x}{x^3}\right] = \frac{-1 + 0}{6}$$

$$\lim_{x \to 0} \left[\frac{\sin x - \cos x}{x^3}\right] = \frac{-1}{6}$$