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L.M.S Assignment II.

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The parametric equations of a curve are as given in eqns (I) & (II)

$$x = \cos t + t \sin t \quad \text{——— (I)}$$

$$y = \sin t - t \cos t \quad \text{——— (II)}$$

In terms of t, determine:

(I) an expression for the radius of curvature (R), and
(II) expressions for the co-ordinates (h, k) of the centre of curvature

Soln.

$$x = \cos t + t \sin t$$

$$\frac{dx}{dt} = -\sin t + t\cos t + \sin t$$

$$\frac{dx}{dt} = t\cos t$$

$$y = \sin t - t\cos t$$

$$\frac{dy}{dt} = \cos t + t\sin t - \cos t$$

$$\frac{dy}{dt} = t\sin t$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$\frac{dy}{dx} = t\sin t \times \frac{1}{t\cos t}$$

$$\frac{dy}{dx} = \frac{\sin t}{\cos t}$$

$$\therefore \frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{\sin t}{\cos t}\right) \times \frac{dt}{dx}$$

$$\frac{d^2y}{dx^2} = \frac{(\cos t)(\cos t) - (\sin t)(-\sin t)}{\cos^2 t} \times \frac{1}{t\cos t}$$

$$\frac{d^2y}{dx^2} = \frac{\cos^2 t + \sin^2 t}{\cos^2 t} \times \frac{1}{t\cos t}$$

Using trig., we know $\sin^2\theta + \cos^2\theta = 1$

$\therefore$

$$\frac{d^2y}{dx^2} = \frac{1}{t\cos^3 t}$$

(I) Radius of Curvature (R)

$$R = \frac{\left(1 + \left(\frac{dy}{dx}\right)^2\right)^{3/2}}{\frac{d^2y}{dx^2}}$$

$$R = \left(1 + \left(\frac{dy}{dx}\right)^2\right)^{3/2} \times \frac{dx^2}{d^2y}$$

$$R = \left(1 + \left(\frac{\sin t}{\cos t}\right)^2\right)^{3/2} \times t\cos^3 t$$

$$R = \left(1 + \frac{\sin^2 t}{\cos^2 t}\right)^{3/2} \times t\cos^3 t$$

$$R = \left(\frac{\cos^2 t + \sin^2 t}{\cos^2 t}\right)^{3/2} \times t\cos^3 t$$

From trig., we know $\sin^2 x + \cos^2 x = 1$

$\therefore$

$$R = \sqrt{\left(\frac{1}{\cos^2 t}\right)^3} \times t\cos^3 t$$

$$R = \frac{1}{\sqrt{(\cos^2 t)^3}} \times t\cos^3 t$$

$$R = \frac{1}{\cos^2 t \times \sqrt{\cos^2 t}} \times t\cos^3 t$$

$$R = \frac{1}{\cancel{\cos^3 t}} \times t\cancel{\cos^3 t}$$

$$R = t$$

(II) Coordinates of centre of Curvature (h, k)

$$h = x_1 - r\sin\theta$$

$$k = y_1 + r\cos\theta$$

$x_1 = \cos t + t\sin t$ $\qquad$ $r = t$

$y_1 = \sin t - t\cos t$ $\qquad$ $\theta = t$

$h = \cos t + t\sin t - t\sin t$

$h = \cos t$

$K = \sin t - t\cos t + t\cos t$

$K = \sin t$

$\therefore (h, K) = (\cos t, \sin t)$