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ENG 281

The parametric equations of a curve as given in eqn (1) and (2)

$$x = \cos t + t\sin t$$

$$y = \sin t - t\cos t$$

In terms of t, determine

i. An expression for the radius of curvature (R), and

ii. Expressions for the coordinates (h, k) of the centre of curvature

i. $$R = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$

$\frac{dy}{dx} = ?$

$\frac{d^2y}{dx^2} = ?$

$$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$$

$y = \sin t - t\cos t$

$$\frac{dy}{dt} = \cos t - [-t\sin t + \cos t]$$

$$= \cos t + t\sin t - \cos t$$

$$\frac{dy}{dt} = t\sin t$$

$x = \cos t + t\sin t$

$$\frac{dx}{dt} = -\sin t + [t\cos t + \sin t]$$

$$= -\sin t + t\cos t + \sin t$$

$$\frac{dx}{dt} = t\cos t$$

$$\frac{dy}{dx} = t\sin t \div t\cos t$$

$$\frac{dy}{dx} = \frac{t\sin t}{t\cos t} = \frac{\sin t}{\cos t}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dt}\left(\frac{dy}{dx}\right)\frac{dt}{dx}$$

$$= \frac{v\frac{dy}{dx} - u\frac{dv}{dx}}{v^2} \times \frac{dt}{dx}$$

$$= \frac{\cos t(\cos t) - \sin t(-\sin t)}{(\cos t)^2} \times \frac{1}{t\cos t}$$

$$= \frac{\cos^2 t + \sin^2 t}{\cos^2 t} \times \frac{1}{t\cos t}$$

$$\frac{d^2y}{dx^2} = \frac{1}{t\cos^3 t}$$

$$R = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$

$$= \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2} \div \frac{d^2y}{dx^2}$$

$$= \left[1 + \left(\frac{\sin t}{\cos t}\right)^2\right]^{3/2} \div \frac{1}{t\cos^3 t}$$

$$= \left[\frac{1}{1} + \frac{\sin^2 t}{\cos^2 t}\right]^{3/2} \times t\cos^3 t$$

$$= \left[\frac{\cos^2 t + \sin^2 t}{\cos^2 t}\right]^{3/2} \times t\cos^3 t$$

$$= \left(\frac{1}{\cos^2 t}\right)^{3/2} \times t\cos^3 t$$

$$= \frac{1}{(\cos^2 t)^{3/2}} \times t\cos^3 t$$

$$= \frac{t\cos^3 t}{\cos^3 t}$$

$$R = t$$

ii. Centre of curvature

$h = x - R\sin t$ ----①

$k = y + R\cos t$ ----②

$x = \cos t + t\sin t$

$y = \sin t - t\cos t$

$R = t$, $t = t$

sub x, R, t into eqn ① and y, R, t into eqn ②

$h = (\cos t + t\sin t) - t\sin t$

$h = \cos t$

$k = (\sin t - t\cos t) + t\cos t$

$k = \sin t$

$(h, k) = (\cos t, \sin t)$