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Engineering Mathematics Assignment .
Solution

1] Evaluate the following limits of function .

$$\lim_{x \to \pi/2} \left[\frac{(x^2 - \pi/4) \sin(\cos x)}{x - \pi/2} \right]$$

Substituting $\pi/2$ for x .

$$= \frac{((\pi/2)^2 - \pi/4) \sin(\cos \pi/2)}{\pi/2 - \pi/2}$$

$$= \frac{(\pi^2/4 - \pi/4) \times 0}{0}$$

$$= \frac{0}{0} \quad \left[\begin{array}{l}\text{Undefined} \\ \text{or Indeterminate}\end{array}\right].$$

Since L'hospitals rule is applied when an indeterminate form is then we differentiate the numerator & denominator by the function

For Numerator.

Let $u = x^2 - \pi/4$

$v = \sin(\cos x)$.

$a = \cos x$

$v = \sin a$.

$\therefore \frac{da}{dx} = -\sin x$, $\frac{dv}{da} = \cos a$.

$\therefore \frac{dv}{dx} = \frac{dv}{da} \times \frac{da}{dv}$

$\frac{dv}{da} = \cos ax - \sin x = -\sin x \, [\cos(\cos x)]$

$\frac{du}{dx} = 2x$

$$\frac{dy}{dx} = V\frac{du}{dx} + U\frac{dv}{dx}.$$

$$\frac{dy}{dx} = \sin(\cos x)[2x] + x^2 . \pi/4 .$$

$$\frac{dy}{dx} = (-\sin x(\cos(\cos x)))$$

For Denominator $\Rightarrow \frac{dy}{dx} = 1.$

$$\lim_{x \to \pi/2} (2x \sin\cos x + x^2 - \pi/4 \ (-\sin x \cos(\cos x)))$$

Substituting $\pi/2 \to x$.

as $x \to \pi/2$; $180/2 \Rightarrow 90$

$$= \frac{\cancel{2}(\pi)}{\cancel{2}} \sin\cos 90 + \left(\frac{\pi}{2}\right)^2 - \frac{\pi}{4}$$

$$(-\sin 90 \cos(\cos 90))$$

$$= 0 \times \pi + \frac{\pi^2}{4} - \frac{\pi}{4} \times (-1 \times 1).$$

$$= \frac{-\pi^2}{4} + \frac{\pi}{4} \Rightarrow \frac{\pi - \pi^2}{4}.$$

$$= \frac{\pi(1-\pi)}{4.} //$$

$$\therefore \lim_{x \to \frac{\pi}{2}} \left[\frac{(x^2 - \pi/4)\sin(\cos x)}{x - \pi/2.}\right]$$

$$= \frac{\pi(1-\pi)}{4.}$$

1b] $\lim_{x \to \pi/2} \ln \left[\exp \frac{(3x^2+2x-1)}{x+1} \right]$

By Factorization

$\frac{3x^2+3x-x-1}{x+1} \Rightarrow \frac{3x(x+1)-1(x+1)}{x+1}$

$= \frac{(3x-1)\cancel{(x+1)}}{\cancel{(x+1)}}$

$\lim_{x \to \pi/2} \ln [\exp(3x-1)]$

$\ln(\exp((3(\pi/2)-1))$

Since exp is anti ln is anti log a

$\therefore \Rightarrow \frac{3}{2}\pi - \frac{2}{2} \Rightarrow \frac{3\pi-2}{2}$

$\lim_{x \to \pi/2} \ln \left[\exp \frac{(3x^2+2x-1)}{x+1} \right]$

$\Rightarrow \frac{3\pi-2}{2}.$

1c] $\lim_{x \to 2+\sqrt{3}} \cos \left[\sin^{-1} \frac{(x-2)}{(x-\sqrt{3})} \right]$

Substituting $2+\sqrt{3}$ for x.

$= \cos \left[\sin^{-1} \frac{(2+\sqrt{3}-2)}{(2+\sqrt{3}-\sqrt{3}} \right]$

$= \cos(\sin^{-1} \sqrt{3}/2)$

$= \cos 60$

$= 1/2.$

$\therefore \lim_{x \to 2+\sqrt{3}} \cos \left[\sin^{-1} \frac{(x-2)}{(x-\sqrt{3})} \right] = \frac{1}{2}.$

1d] $\lim_{x \to 4} \left[\frac{x^2-8x+16}{x^2-5x+4} \right]$

By Factorizing.

$$\frac{x^2-4x-4x+16}{x^2-x-4x+4} \Rightarrow \frac{x(x-4)-4(x-4)}{x(x-1)-4(x-1)}$$

$$= \frac{\cancel{(x-4)}(x-4)}{\cancel{(x-4)}(x-1)} \Rightarrow \frac{x-4}{x-1}.$$

where $x \to 4$.

$$\therefore \frac{4-4}{4-1} = \frac{0}{3} = 0.$$

$$\therefore \lim_{x \to 4} \left(\frac{x^2-8x+16}{x^2-5x+4} \right) = 0$$

(2) Determine wheater each of the following series is convergent.

(a) $\frac{2}{2\times3} + \frac{2}{3\times4} + \frac{2}{4\times5} + \frac{2}{5\times6} + \cdots$

Using D'Alembert's ratio

$$U_n = \frac{2}{(n+1)(n+2)}$$

$$U_{n+1} = \frac{2}{(n+2)(n+3)}.$$

$$\frac{U_{n+1}}{U_n} = \frac{\cancel{2}}{\cancel{(n+2)}(n+3)} \times \frac{(n+1)\cancel{(n+2)}}{\cancel{2}}$$

$$\frac{U_{n+1}}{U_n} = \frac{n+1}{n+3}.$$

Dividing through by the highest power of n (n) [Through numerator and denominator]

$$\lim_{n\to\infty} \left[\frac{n/n + 1/n}{n/n + 3/n}\right] = \frac{1 + 1/n}{1 + 3/n}.$$

as $n \to \infty$

$1/n \to 0$

$3/n \to 0$

$$\frac{1+0}{1+3} = \frac{1}{1} = 1.$$

Since $\frac{U_{n+1}}{U_n} = 1$

The series is either divergent or convergent

For testing further, Using Test 1

$$\lim_{n\to\infty} U_n = \frac{2}{n^2+2n+n+2} \Rightarrow \frac{2}{n^2+3n+2}$$

Divide through by n^2 the highest power of n (through numerator and denominator).

$$\lim_{n\to\infty} U_n = \frac{2/n^2}{n^2/n^2 + 3n/n^2 + 2/n^2} \Rightarrow \frac{2/n^2}{1 + 3/n + 2/n^2}$$

as $n \to \infty$.

$2/n^2 = 0$, $3/n \to 0$.

$$= \frac{0}{1+0+0} = \frac{0}{1} = 0$$

Since $\lim_{n\to\infty} U_n = 0$.

Testing further Using Comparison Test since n either convergent or divergent

$$\therefore \frac{\cancel{2}}{\cancel{2}\times 3} + \frac{\cancel{2}}{3\times \cancel{4}_2} + \frac{\cancel{2}}{{}_2\cancel{4}\times 5} + \frac{\cancel{2}}{5\times \cancel{6}_3} + \cdots$$

$$\frac{1}{3} + \frac{1}{6} + \frac{1}{10} + \frac{1}{15} + \cdots$$

$\frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \frac{1}{4^p} + \cdots$ (a)

where $p = 2$.

$= \frac{1}{1} + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \cdots$ (b)

Using Comparison test.

$\frac{1}{3} < 1$; $\frac{1}{6} < \frac{1}{4}$.

and the series (b) is known to converge then,

$\frac{2}{2\times3} + \frac{2}{3\times4} + \frac{2}{4\times5} + \frac{2}{5\times6} + \cdots$

is convergent

1b) $\frac{2}{1^2} + \frac{2}{2^2} + \frac{2}{3^2} + \frac{2}{4^2} + \cdots$

Using D' Alemberts ratio:

$U_{n+1} = \frac{2}{(n+1)^2} = \frac{2}{n^2+2n+1}$.

$\frac{U_{n+1}}{U_n} = \frac{2}{n^2+2n+1} \times \frac{n^2}{2} \Rightarrow \frac{n^2}{n^2+2n+1}$.

$\lim_{n\to\infty} \frac{U_{n+1}}{U_n} = \frac{n^2}{n^2+2n+1}$.

Divide through by the highest Power of n (n^2)

$= \frac{n}{n^2/n^2 + 2n/n^2 + 1/n^2}$

$\lim_{n\to\infty} \frac{U_{n+1}}{U_n} = \frac{1}{1+2/n+1/n^2}$

$n\to\infty$ $2/n, \frac{1}{n^2} \to 0$.

$= \frac{1}{1+0+0} = 1/1 = 1$.

Since $\lim_{n\to\infty} \frac{U_{n+1}}{U_r} = 1$.

The series is either convergent or divergent so testing further
Using Test 1

$$\lim_{n\to\infty} U_n = \frac{n^2}{n^2} \cdot \frac{2}{n^2}$$

dividing through by n^2

$$\lim_{n\to\infty} U_n = \frac{2/n^2}{n^2/n^2} \cdot \frac{2/n^2}{1} \Rightarrow \frac{2}{n^2}$$

as $n\to\infty$, then $2/n^2 \to 0$.

The Series may be Convergent or divergent.
Using Comparison

$$\frac{2}{1} + \frac{\cancel{2}}{\cancel{4}} + \frac{\cancel{2}}{9} + \frac{\cancel{2}}{\cancel{16}} + \cdots$$

$$\frac{2}{1} + \frac{1}{2} + \frac{2}{9} + \frac{1}{8} + \cdots$$

$$\frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \frac{1}{4^p} + \cdots \quad (a)$$

$$\frac{1}{1} + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \cdots \quad (b).$$

Comparing them

$2 > 1$, $1/2 > 1/4$.

and the series is known to be ~~convergent~~ divergent

(2c) $U_n = \frac{1+2n^2}{1+n^2}$.

$$\lim_{n\to\infty} U_n = \frac{1+2n^2}{1+n^2}$$

Divide through by n^2

$$\lim_{n\to\infty} U_n = \frac{1/n^2 + 2n^2/n^2}{1/n^2 + \cancel{n^2}/n^2} = \frac{1/n^2 + 2}{1/n^2 + 1}.$$

as $n \to \infty$ $\quad 1/n^2 \to 0$.

$$= \frac{2+0}{1+0} = \frac{2}{1}.$$

Since $\lim_{n \to \infty} U_n \neq 0$.

The series is Divergent.

(3) Find the range of values of x for which the series below is absolutely convergent.

$$\frac{x}{27} + \frac{x^2}{125} + \cdots + \frac{x^n}{(2n+1)^3}$$

Using D'alembert's ratio.

$$U_n = \frac{x^n}{(2n+1)^3}.$$

$$U_{n+1} = \frac{x^{n+1}}{(2n+3)^3}$$

$$\frac{U_{n+1}}{U_n} = \frac{x^{n+1}}{(2n+3)^3} \times \frac{(2n+1)^3}{x^n}$$

$$\frac{U_{n+1}}{U_n} = \frac{x(2n+1)^3}{(2n+3)^3}$$

$$\frac{U_{n+1}}{U_n} = \frac{x\left[(2n)^3 + 3(2n)^2(1) + 3(2n)(1)^2 + (1)^3\right]}{(2n)^3 + 3(2n)^2(3)^1 + 3(2n)(3)^2 + (3)^3}$$

$$\frac{U_{n+1}}{U_n} = \frac{x(8n^3 + 12n^2 + 6n + 1)}{8n^3 + 36n^2 + 58n + 27}.$$

$$\frac{U_{n+1}}{U_n} = \frac{8n^3x + 12n^2x + 6nx + x}{8n^3 + 36n^2 + 54n + 27}$$

Divide through by n^3 the highest power of n

$$\frac{U_{n+1}}{U_n} = \frac{8n^3x/n^3 + 12n^2x/n^3 + 6nx/n^3 + x/n^3}{8n^3/n^3 + 36n^2/n^3 + 54n/n^3 + 27/n^3}$$

$$\frac{U_{n+1}}{U_n} = \frac{8x + 12x/n + 6x/n^2 + x/n^3}{8 + 36/n + 54/n^2 + 27/n^3}$$

as $n \to \infty$

$12x/n \to 0$, $6x/n^2 \to 0$, $x/n^3 \to 0$.

$36/n \to 0$, $54/n^2 \to 0$, $27/n^3 \to 0$.

$$\therefore \lim_{n \to \infty} \frac{U_{n+1}}{U_n} = \frac{8x + 0 + 0 + 0}{8 + 0 + 0} = \frac{\cancel{8}x}{\cancel{8}}$$

Since the series is convergent.

$$\lim_{n \to \infty} \frac{U_{n+1}}{U_n} < 1.$$

$\therefore$ $x < 1$ is the range of values of x for which the series is absolutely convergent

4) Evaluate Using L'hopital Rule.

$$\lim_{x \to 0} \left[\frac{\sin x - \cos x}{x^3} \right]$$

Substituting 0 for x as $x \to 0$.

$$\frac{\sin 0 - \cos 0}{0^3} = \frac{0-1}{0} = \frac{-1}{0} = -1$$

[Indeterminate or Undefined].

$$\frac{dy}{dx} \Rightarrow \frac{\cos x + \sin x}{3x^3} \Rightarrow \lim_{x \to 0} \left[\frac{\cos x + \sin x}{3x^2} \right]$$

Substituting x for x as $x \to 0$. [deteterminate or Undefined]

$\frac{d^2y}{dx^2} \Rightarrow \frac{-\sin x + \cos x}{6x}$

$= \lim_{x \to 0} \left[\frac{\cos x - \sin x}{6x} \right]$

Subst. 0 for x as $x \to 0$.

$= \frac{\cos 0 - \sin 0}{6(0)} = \frac{1-0}{0} = \frac{1}{0} = 1.$

[Indeterminate or Undefined]

$\frac{d^2y}{dx^3} \to \frac{-\sin x - \cos x}{6.}$

$\lim_{x \to 0} \left(\frac{-\sin x - \cos x}{6} \right)$

Subst. 0 for x as $x \to 0$..

$= \frac{-\sin 0 - \cos 0}{6} = \frac{0-1}{6} = -1/6$

$\therefore \lim_{x \to \infty} \frac{\sin x - \cos x}{x^3} = -1/6.$