EUG 351
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MECHANICAL ENGR
13/ENG06/008

1) $d^2y/dx^2 - dy/dx - 2y = 8$

$m^2 - m - 2 = 0$

$m_1 = 2 \quad m_2 = -1$

$y = Ae^{2x} + Be^{-x}$

$P_i : y = c$

$dy/dx = 0$

$d^2y/dx^2 = 0$

$0 - 0 - 2c = 8$

$-2c = 8 \Rightarrow c = -4$

$P_i : y = -4$

G.S : $y = Ae^{2x} + Be^{-x} - 4$

3) $d^2y/dx^2 - 4y = 10e^{3x}$

$m^2 - 4 = 0$

$m^2 = 4$

$m = \pm\sqrt{4} \Rightarrow \pm 2j$

$y = C\cosh 2x + D\sinh 2x$

$P_i : y = Ce^{3x}$

$dy/dx = 3Ce^{3x}$

$d^2y/dx^2 = 9Ce^{3x}$

$9Ce^{3x} - 4(Ce^{3x}) = 10e^{3x}$

$5Ce^{3x} = 10e^{3x}$

$C = \dfrac{10e^{3x}}{3e^{3x}}$

G.S = $C\cosh 2x + D\sinh 2x$

$d^2y/dx^2 + 2dy/dx + y = e^{-2x}$

$m + 2m + 1 = 0$

$\dfrac{-2 \pm \sqrt{(2)^2 - 4 \times 1 \times 1}}{2 \times 1} = -\dfrac{2}{2} = -1$

$m = -1$

$y = e^{-x}(A + Bx)$

P.I : $y = Ce^{-2x}$

$dy/dx = -2Ce^{-2x}$

$d^2y/dx^2 = 4Ce^{-2x}$

$4Ce^{-2x} - 2(2Ce^{-2x}) + Ce^{-2x} = e^{-2x}$

$y = e^{-2x}$

G.S $\Rightarrow y = e^{-x}(A + B + e^{-2x})$

4) $d^2y/dx^2 + 25y = 5x^2 + x$

$m^2 + 25 = 0$

$m^2 = -25$

$m = \pm\sqrt{25}$

$m = \pm 5j$

$y = C\cosh 5x + D\sinh 5x$

P.I : $y = Cx^2 + Dx + E$

$dy/dx = 2Cx + D$

$d^2y/dx = 2C$

$25(Cx^2 + Dx + E) = 6x^2 + x$

$25C = 5$ Comparing coeffs.

$25D = 1$; $D = 1/25$

$25E + 2C = 0$

$25E + 2(1/5) = 0$

$25E = -2/5$

$E = -2/125$

$y = 1/5x^2 + 1/25x - 2/125$

G.S $\Rightarrow y = C\cos 5x\ D\sin 5x + 1/5x^2 + 1/25x - 2/125$

$d^2y/dx^2 - 2dy/dx + y = 4\sin x$

$m^2 - 2m + 1$

$= \pm\dfrac{\sqrt{(-2)^2 - 4\times1\times1}}{2\times1} = 2/2 = 1$

$m = 1$

$y = e^x(A + Bx)$

$dy/dx = -C\sin x + D\cos x$

$d^2y/dx^2 = C\cos x - D\sin x$

$-C\cos x - D\sin x - 2C - C\sin x + D\cos x) + (C\cos x + D\sin x) = 4\sin x$

$-C\cos x - D\sin x + 2C\sin x - 2D\cos x + C\cos x + D\sin x = 4\sin x$

$= -2D\cos x + 2C\sin x = 4\sin x$

comparing coeffs

$-2D = 0 \qquad 2C = 4$

$D = -0/2 = 0 \qquad C = 4/2 = 2$

$y = 2\cos x + 0\sin x$

$y = 2\cos x$

G.S : $y = e^x(A + Bx)$

6) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$ given that $x = 0$

$y = 1$ and $dy/dx = -2$

$m^2 + 4m + 5$

$\frac{-4 \pm \sqrt{(4)^2 - 4\times1\times5}}{2\times1} = \frac{-4 \pm i}{2} = -2 \pm i$

$y = e^{-2x}(C\cos 2x + D\sin 2x)$

P.I : $y = Ce^{-2x}$

$dy/dx = -2Ce^{-2x}$

$d^2y/dx = 4Ce^{-2x}$

$4Ce^{-2x} + 4C\cdot 2e^{-2x}) + 5(Ce^{-2x}) = 2e^{-2x}$

$4Ce^{-2x} - 9Ce^{-2x} + 5Ce^{-2x} = 2e^{-2x}$

$4C + x - 8C = 2$

$C = 2, \; y = 2e^{-2x}$

G.S : $y = e^{-2x}(\cos 2x + D\sin 2x) + 2e^{-2x}$

When $x = 0$

$y = e^{-2(0)}(\cos 2(0) + D\sin(2(0))) + 2e^{-2(0)}$

$1 = C + 2 \quad \therefore C = -1$

$dy/dx = e^{-2x}(-2\sin 2x + 2D\cos 2x) + (C\cos 2x + D\sin 2x) - 4e^{-2x}$

at $x = 0$

$D = -1$

G.S : $y = e^{-2x}(-C\cos 2x - D\sin 2x)$

7) $3d^2y/dx^2 - 2dy/dx - y = 2x - 3$

$3m^2 - 2m - 1 = 0$

$\frac{2 \pm \sqrt{(-2)^2 - 4 \times 3 \times 1}}{6} = \frac{2 \pm \sqrt{4+12}}{6}$

$= \frac{2 \pm \sqrt{16}}{6} = \frac{2 \pm 4}{6} \Rightarrow 1 \text{ or } -1/3$

$m_1 = 1 \; ; \; m_2 = -1/3$

$\therefore y = Ae^x + Be^{-1/3}$

$P.I : y = cx + D$

$dy/dx = c$

$d^2y/dx = 0$

$3(0) - 2x - (cx + D) 2x - 3$

$-2c - cx - D = 2x - 3$

Comparing coefficients

$c = -2$

$-2c - D = -3$

$-2(-2) - D = -3$

$4 - D = -3$

$D = 4 + 3 = 7$

$y = -2x + 7$

$G.S : y = Ae^x + Be^{-1/3} - 2x + 7$

8) $d^2y/dx^2 - 6dy/dx + 9y = 8e^{4x}$.

$m^2 - 6m + 4 = 0$

$x_1 = 4x_2 - 2$

$y = Ae^{4x} + Be^{2x} + e^{4x}$