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ENG 281
MECH. ENG
16/ENG06/074

Solution

$$\lim_{x \to \pi/2} \left[\frac{(x^2 - \pi/4) \sin(\cos x)}{x - \pi/2} \right]$$

$$= \left[\frac{\left(\left(\frac{\pi}{2}\right)^2 - \frac{\pi}{4} \right) \sin\left(\cos \pi/2\right)}{\pi/2 - \pi/2} \right]$$

$$= \left[\frac{\left(\left(\frac{\pi}{2}\right)^2 - \frac{\pi}{4} \right) \sin\left(\cos \pi/2\right)}{\pi/2 - \pi/2} \right] = \frac{\left(\frac{\pi^2}{4} - \frac{\pi}{4} \right) \sin\left(\cos \frac{\pi}{2}\right)}{0}$$

ie indeterminant

$\therefore$ using L'Hopitals Law, dy/dx of the numerator $u\,dv/dx + v\,du/dx$

$dy/dx =$ let $u = x^2 - \pi/4$

and $v = \sin(\cos x)$

$\frac{dy}{dx} = 2x$ $\quad \frac{dv}{dx} = ?$

$d/dx \sin(\cos x) =$ let $\cos x = w$

$v = \sin w$

$dv/dw = \cos w$ $\quad \therefore dw/dx = -\sin x$

$dv/dx = dv/dw \times dw/dx = -\sin x \cos(\cos x)$

$$= \frac{(x^2 - \pi/4) \times -\sin x \cos(\cos x) + \sin(\cos x)(2x)}{1}$$

$= (\pi/2)^2 - \pi/4 \times -\sin 90 \cos(\cos 90) + \sin \cos 90 \times 2(\pi/2)$

$= (\pi^2/4 - \pi/4) \times -1 + 0 \times \pi$

$= -\pi^2/4 + \pi/4 \quad ; \quad = \pi/4 - \pi^2/4$

$\therefore$

$$\lim_{x = \pi/2} \left[\frac{(x^2 - \pi/4) \sin(\cos x)}{x - \pi/2} \right] = \frac{\pi(1 - \pi)}{4}$$

0

ie indeterminant

$\therefore$ Using L'Hopital's Law, dy/dx of the numerator: $u\,dv/dx + v\,du/dx$

dy/dx: let $u = x^2 - \pi/4$

and $v = \sin(\cos x)$

$\frac{du}{dx} = 2x$ $\qquad$ $\frac{dv}{dx} = ?$

$d/dx \sin(\cos x)$: let $\cos x = \omega$

$v = \sin \omega$

$dv/d\omega = \cos \omega$ $\qquad$ $\therefore$ $d\omega/dx = -\sin x$

$dv/dx = dv/d\omega \times d\omega/dx = -\sin x \cos(\cos x)$

$= (x^2 - \pi/4) \times \underline{-\sin x\,\cos(\cos x)} + \sin(\cos x)(2x)$

b) $\lim \ln \left[\exp \left(\frac{3x^2 + 2x - 1}{x+1} \right) \right]$

$= \lim_{x \to \pi/2} \ln \left(\exp \left[\frac{(3x-1)(x+1)}{x+1^2} \right] \right)$

$= \lim_{x \to \pi/2} \ln \left(\exp (3x - 1) \right)$

$= \lim_{x \to \pi/2} (3x - 1) = 3(\pi/2) - 1$

$= 3\pi/2 - 1 = \frac{3\pi}{2} - \frac{1}{1}$

$\therefore \lim_{x \to \pi/2} \ln \left[\exp \frac{(3x^2 + 2x - 1)}{x+1} \right] = \frac{3\pi - 2}{2}$

c) $\lim_{x \to 2+\sqrt{3}} \cos \left(\sin^{-1} \frac{(x-2)}{(x-\sqrt{3})} \right)$

$= \lim_{x \to 2+\sqrt{3}} \cos \left[\sin^{-1} \frac{(2+\sqrt{3} - 2)}{(2 + \sqrt{3} - \sqrt{3})} \right]$

$= \cos \left[\sin^{-1} \left[\frac{\sqrt{3}}{2} \right] \right]$

$= \cos \left(\sin^{-1} (0.8660) \right] \Rightarrow \cos 60^\circ$

$= 1/2$

D $\lim_{x \to 4} \left[\frac{x^2 - 8x + 16}{x^2 - 5x + 4} \right]$

$\Rightarrow \lim_{x \to 4} \left[\frac{(x-4)(x-4)}{(x-4)(x-1)} \right]$

$= \lim_{x \to 4} \left[\frac{x-4}{x-1} \right]$

$= \frac{4-4}{4-1} = \frac{0}{3} = 0$

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2A) $u_n = \frac{2}{(n+1)(n+2)}$

$u_{n+1} = \frac{2}{(n+2)(n+3)}$

ratio $= \frac{u_{n+1}}{u_n} = \frac{2}{(n+2)(n+3)} \times \frac{(n+1)(n+2)}{2}$

$\frac{u_{n+1}}{u_n} = \frac{n+1}{n+3}$

$\therefore \lim_{n \to \infty} \frac{u_{n+1}}{u_n} = \lim_{n \to \infty} \frac{n+1}{n+3}$

$= \frac{n/n + 1/n}{n/n + 3/n} = \frac{1 + 1/n}{1 + 3/n} = \frac{1+0}{1+0} = 1$

$\therefore \lim_{n \to \infty} \frac{u_{n+1}}{u_n}$ $\therefore$ the series is inconclusive

B) $\left[1/1^r + 1/2^r + 1/3^r + 1/4^r + \cdots 1/n^r \right] = \sum_{n=1}^{\infty} 1/n^p$

$\Rightarrow \left[2/1^2 + 2/2^2 + 2/3^2 + 2/4^2 + \cdots 2/n^2 \right] = \sum_{n=1}^{\infty} 2/n^2$

$p = 2$

$\therefore p > 1$, the series will converge

3) $U_n = \frac{x^n}{(2n+1)^3}$, $u_{n+1} = \frac{x^{n+1}}{(2n+2)^3}$

$\lim \frac{u_{n+1}}{u_n} = \frac{x^{n+1}}{(2n+2)^3} \times \frac{(2n+1)^3}{x^n}$

$\frac{x(2n+1)^3}{(2n+2)^3} = \frac{8n^3 + 12n^2 + n + 1}{8n^3 + 24n^2 + 24n + 8}$

divide through by n^3

$= \frac{(8 + 2/n + 4/n^2 + 1/n^3)}{(8 + 12/n + 24/n^2 + 8/n^3)}$

$\infty \rightarrow \infty$

$1/n \rightarrow 0$

$\frac{8x}{8} \geq x - 1$

$x < 1$

4) $\lim_{x \to 0} \left[\frac{\sin x - \cos x}{x^3} \right]$

by using L' Hopital's rule

$$y = \left[\frac{\sin x - \cos x}{x^3} \right]$$

$$\frac{dy}{dx} = \left[\frac{\cos x + \sin x}{3x^2} \right]$$

$$\frac{d^2y}{dx^2} = \frac{-\sin x + \cos x}{6x}$$

$$\frac{d^2y}{dx^3} = \frac{-\cos x - \sin x}{6}$$

$$\lim = \frac{-\cos 0 - \cos 0}{6} = \frac{\{1 - 0\}}{6} = \frac{-1}{6}$$

$$\lim_{x \to 0} \left[\frac{\sin x - \cos x}{x^3} \right] = -\frac{1}{6}$$