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MATRIC NO: 15/ENG07/014
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ENG 381 ASSIGNMENT 1

1. $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$

Solution

Assume homogenity

$y'' - y' - 2y = 0$

$y = e^{kx}$

$y' = ke^{kx}$

$y'' = k^2 e^{kx}$

Substitute the above into

$y'' - y' - 2y = 0.$

$k^2 e^{kx} - ke^{kx} - 2e^{kx} = 0$

$e^{kx}(k^2 - k - 2) = 0$

$k^2 - k - 2 = 0$

$k^2 - 2k + k - 2 = 0$

$(k^2 - 2k) + (k - 2) = 0$

$k(k-2) + 1(k-2) = 0$

$(k+1)(k-2) = 0$

$k + 1 = 0$

$k - 2 = 0$

$k_1 = -1, \quad k_2 = 2$

$y_1 = e^{k_1 x} = e^{-x}$

$y_2 = e^{k_2 x} = e^{2x}$

$y = C_1 y_1 + C_2 y_2$

$y = C_1 e^{-x} + C_2 e^{2x}$

$y' = 0$

$y'' = 0$

$y_p = K$ (assumption)

substitute into $y'' - y' - 2y = 8$

$0 - 0 - 2K = 8$

$\frac{-\cancel{2}K}{-\cancel{2}} = \frac{8}{-2}$; $K = -4$

$\therefore y_s = C_1 e^{-x} + C_2 e^{2x} + c$
$y_s = C_1 e^{-x} + C_2 e^{2x} - 4$

2. $\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

solution

$y'' - 4y = 0$
$y = e^{kx}$
$y' = ke^{kx}$
$y'' = k^2 e^{kx}$
$k^2 e^{kx} - 4e^{kx} = 0$
$e^{kx}(k^2 - 4) = 0$
$k^2 - 4 = 0$
$k^2 = 4$
$k = \pm\sqrt{4}$
$k_1 = +2 \quad , \quad k_2 = -2$
$y_1 = e^{k_1 x} = e^{2x}$
$y_2 = e^{k_2 x} = e^{-2x}$
$y = C_1 y_1 + C_2 y_2$
$y = C_1 e^{2x} + C_2 e^{-2x}$
$y_p = Ae^{3x}$
$y' = 3Ae^{3x}$
$y'' = 9Ae^{3x}$
Substitute into $y'' - 4y = 10e^{3x}$
$9Ae^{3x} - 4Ae^{3x} = 10e^{3x}$
$e^{3x}(9A - 4A) = 10e^{3x}$
$5A = 10 \quad ; \quad A = 2$
$\therefore y_s = C_1 e^{2x} + C_2 e^{-2x} + Ae^{3x}$
$y_s = C_1 e^{2x} + C_2 e^{-2x} + 2e^{3x}$

3. $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}$

Solution

$y'' + 2y' + y = 0$
$y = e^{kx}$
$y' = ke^{kx}$
$y'' = k^2 e^{kx}$

Substitute into $y'' + 2y' + y = 0$

$k^2e^{kx} + 2ke^{kx} + e^{kx} = 0$

$e^{kx}(k^2 + 2k + 1) = 0$

$k^2 + 2k + 1 = 0$

$(k^2 + k) + (k + 1) = 0$

$k(k+1) + 1(k+1) = 0$

$(k+1)(k+1) = 0$

$k + 1 = 0$

$k + 1 = 0$

$k_1 = -1, \ k_2 = -1$

$y_1 = e^{k_1x} = e^{-x}$

$y_2 = e^{k_2x} = e^{-x}$

$y = C_1y_1 + C_2y_2$

$y = C_1e^{-x} + C_2e^{-x}$

$y_p = Ae^{-2x}$

$y' = -2Ae^{-2x}$

$y'' = 4Ae^{-2x}$

Substitute into $y'' + 2y' + y = e^{-2x}$

$4Ae^{-2x} + 2(-2Ae^{-2x}) + Ae^{-2x} = e^{-2x}$

$\cancel{4Ae^{-2x}} - \cancel{4Ae^{-2x}} + Ae^{-2x} = e^{-2x}$

$A = 1$

$y_s = C_1e^{-x} + C_2e^{-x} + e^{-2x}$

$y_s = e^{-x}(C_1 + C_2) + e^{-2x}$

4 $\frac{d^2y}{dx^2} + 25y = 5x^2 + x$

Solution

$y'' + 25y = 0$

$y = e^{kx}$

$y' = ke^{kx}$

$y'' = k^2e^{kx}$

$k^2e^{kx} + 25e^{kx} = 0$

$e^{kx}(k^2 + 25) = 0$

$k^2 + 25 = 0$

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$K^2 = -25$

$K = \pm\sqrt{-5}$

$K_1 = 5i \quad , K_2 = -5i$

$y_1 = \cancel{e^x}\, e^{K_1 x} = e^{5ix}$

$y_2 = e^{K_2 x} = e^{-5ix}$

$y = C_1 y_1 + C_2 y_2$

$y = C_1 e^{5xi} + C_2 e^{-5xi}$

$y_p = Ax^2 + Bx + C$

$y' = 2Ax + B$

$y'' = 2A$

Substitute into $y'' + 25y = 5x^2 + x$

$2A + 25(Ax^2 + Bx + C) = 5x^2 + x$

$2A + 25Ax^2 + 25Bx + 25C = 5x^2 + x$

$25Ax^2 = 5x^2$

$A = 5/25 = 1/5$ — ⊛

$25Bx = x$

$B = 1/25$ — ⊛

$2A + 25C = 0$. Where $A = 1/5$

$2(1/5) + 25C = 0$

$25C = -2/5$

multiply both sides by $1/25$

$C = -2/125$

$y_p = Ax^2 Bx + C$ ~~$1/5 x^2 + 1/25$~~

$= 1/5 x^2 + 1/25 x - 2/125$

$y_s = C_1 e^{5xi} + C_2 e^{-5xi} + 1/125(25x^2 + 5x - 2)$

$y_s = C_1 \cos 5t + C_2 \sin 5t + 1/125(25x^2 + 5x - 2)$

5. $\frac{d^2y}{dx^2} - \frac{2dy}{dx} + y = 4\sin x$

Solution.

$y'' - 2y' + y = 0$

$y = e^{kx}$

$y' = ke^{kx}$

$y'' = k^2 e^{kx}$

$k^2 e^{kx} - 2ke^{kx} + e^{kx} = 0$

$e^{kx}(k^2 - 2k + 1) = 0$

$k^2 - 2k + 1 = 0$

$(k^2 - k) - (k - 1) = 0$

$k(k-1) - 1(k-1) = 0$

$(k-1)(k-1) = 0$

$k - 1 = 0$

$k - 1 = 0$

$k_1 = 1, \quad k_2 = 1$

$y_1 = e^{k_1 x} = e^x$

$y_2 = e^{k_2 x} = e^x$

$y = C_1 y_1 + C_2 y_2 = C_1 e^x + C_2 e^x$

$y_p = A\sin x + B\cos x$

$y' = A\cos x - B\sin x$

$y'' = -A\sin x - B\cos x$

Substitute into $y'' - 2y' + y = 0$

$-A\sin x - B\cos x - 2(A\cos x - B\sin x)$

$+ A\sin x + B\cos x = 4\sin x$

$-A\sin x - B\cos x - 2A\cos B + 2\sin x$

$+ A\sin x + B\cos x = 4\sin x$

$(\cancel{-A} + 2B + \cancel{A})\sin x + (\cancel{-B} - 2A + \cancel{B})\cos x$

$= 4\sin x$

$2B\cancel{\sin x} = 4\cancel{\sin x}$

$B = 4/2 = 2$

$-2A\cos x = 0\cos x$

$A = 0$

$y_s = C_1 e^x + C_2 e^x + A\sin x + B\cos x$

$y_s = C_1 e^x + C_2 e^x + 0\sin x + 2\cos x$

$y_s = C_1 e^x + C_2 e^x + 2\cos x$

$y_s = e^x(C_1 + C_2) + 2\cos x$.

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6.) $\frac{d^2y}{dx^2} + \frac{4dy}{dx} + 5y = 2e^{-2x}$

When $x = 0$, $y = 1$, $dy/dx = -2$.

Solution

$y'' + 4y' + 5y = 0$

$y = e^{kx}$

$y' = ke^{kx}$

$y'' = k^2 e^{kx}$

$k^2 e^{kx} + 4ke^{kx} + 5e^{kx} = 0$

$e^{kx}(k^2 + 4k + 5) = 0$

$k^2 + 4k + 5 = 0$

Using $k = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

where $a = 1$, $b = 4$, $c = 5$

$k = \frac{-4 \pm \sqrt{(4)^2 - 4 \times 1 \times 5}}{2 \times 1}$

$k = \frac{-4 \pm \sqrt{16 - 20}}{2}$

$k = \frac{-4 \pm \sqrt{-4}}{2}$

$k = \frac{-4 \pm 2i}{2}$

$k_1 = \frac{-4 + 2i}{2} = \frac{\cancel{2}(-2 + i)}{\cancel{2}}$

$k_1 = -2 + i$, $k_2 = -2 - i$

$y_1 = e^{k_1 x} = e^{(-2+i)x}$

$y_2 = e^{k_2 x} = e^{(-2-i)x}$

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$y_1 = e^{-2x} \cdot e^{ix}$ DATE

$y_2 = e^{2x} \cdot e^{-ix}$

$y = C_1 y_1 + C_2 y_2$

$y = C_1 (e^{-2x} \cdot e^{ix}) + C_2 (e^{-2x} \cdot e^{-ix})$

$y = e^{-2x} [C_1 e^{ix} + C_2 e^{-ix}]$

$y = e^{-2x} [C_1 \cos x + C_2 \sin x]$.

$y_p = Ae^{-2x}$

$y' = -2Ae^{-2x}$

$y'' = 4Ae^{-2x}$

Substitute into $y'' + 4y' + 5y = 2e^{-2x}$

$4Ae^{-2x} + 4(-2Ae^{-2x}) + 5(Ae^{-2x}) = 2e^{-2x}$

$4Ae^{-2x} - 8Ae^{-2x} + 5Ae^{-2x} = 2e^{-2x}$

$\cancel{e^{-2x}}(4A - 8A + 5A) = 2\cancel{e^{-2x}}$

$A = 2$

$\therefore y = e^{-2x} [C_1 \cos x + C_2 \sin x] + 2e^{-2x}$

at $x = 0$, $y = 1$

$1 = e^{-2(0)} [C_1 \cos(0) + C_2 \sin(0)] + 2e^{-2(0)}$

$1 = 1 [C_1 + 0] + 2$

$1 = C_1 + 2$

$1 - 2 = C_1$; $C_1 = -1$

To get C_2

$y' = -2e^{-2x} [-C_1 \sin x + C_2 \cos x] - 4e^{-2x}$

at $x = 0$, $y - 2$.

$-2 = -2e^{-2(0)} [-C_1 \sin(0) + C_2 \cos(0)] - 4e^{-2(0)}$

$-2 = -2[0 + C_2] - 4$

$-2 = -2C_2 - 4$; $-2 + 4 = -2C_2$

$C_2 = -1$

$y = e^{-2x}[C_1 \cos x + C_2 \sin x] + 2e^{-2x}$ DATE

Substitute $C_1 = -1$ and $C_2 = -1$

$y = e^{-2x}[-\cos x - \sin x] + 2e^{-2x}$

$y = e^{-2x}[-\cos x - \sin x + 2]$

$y = e^{-2x}[2 - \cos x - \sin x]$

7) $3\frac{d^2y}{dx^2} - 2\,{}^{dy}/_{dx} - y = 2x - 3$

Solution

$3y'' - 2y' - y = 0$

$y = e^{kx}$

$y' = ke^{kx}$

$y'' = k^2 e^{kx}$

$3k^2 e^{kx} - 2ke^{kx} - e^{kx} = 0$

$e^{kx}(3k^2 - 2k - 1) = 0$

$3k^2 - 2k - 1 = 0$

$(3k^2 - 3k) + (k - 1) = 0$

$3k(k-1) + 1(k-1) = 0$

$(3k+1)(k-1) = 0$

$3k + 1 = 0$; $k - 1 = 0$

$k_1 = -1/3$, $k_2 = 1$

$y_1 = e^{k_1 x} = e^{-1/3 x}$

$y_2 = e^{k_2 x} = e^{x}$

$y = C_1 y_1 + C_2 y_2$

$y = C_1 e^{-1/3x} + C_2 e^{x}$

$y_p = Ax - Bx^0$

$y' = A - 0 + 0$

$y'' = 0$

Substitute into $3y'' - 2y' - y = 2x - 3$ DATE

$0 - 2A - Ax - B = 2x - 3$

$-Ax = 2x$

$A = -2$ ——→ *

$-2A - B = 3$. Where $A = -2$

$-2(-2) - B = -3$

$4 - B = -3$

$4 + 3 = B$

$B = 7$ ——→ *

$y_p = Ax - B = -2x + 7$

$\therefore y_s = C_1 e^{-1/3x} + C_2 e^{x} + y_p$

$\therefore y_s = C_1 e^{-1/3x} + C_2 e^{x} - 2x + 7$

8) $\frac{d^2y}{dx^2} - 6\,{dy}/{dx} + 8y = 8e^{4x}$

Solution

$y'' - 6y' + 8y = 0$

$y = e^{kx}$

$y' = ke^{kx}$

$y'' = k^2 e^{kx}$

$k^2 e^{kx} - 6ke^{kx} + 8e^{kx} = 0$

$e^{kx}(k^2 - 6k + 8) = 0$

$k^2 - 6k + 8 = 0$

$(k^2 - 2k) + (-4k + 8) = 0$

$k(k-2) - 4(k-2) = 0$

$(k-4)(k-2) = 0$

$k - 4 = 0$

$k - 2 = 0$

$k_1 = 4$; $k_2 = 2$.

*

No 8 continuation

$y_1 = e^{k_1 x} = e^{4x}$

$y_2 = e^{k_2 x} = e^{2x}$

$y = c_1 y_1 + c_2 y_2$

$y = c_1 e^{4x} + c_2 e^{2x}$

$y_p = Axe^{4x}$

$y' = A[4xe^{4x} + e^{4x}]$

$y' = 4Axe^{4x} + Ae^{4x}$

$y'' = A[4x(4e^{4x}) + e^{4x}(4) + 4e^{4x}]$

$y'' = A[16xe^{4x} + 4e^{4x} + 4e^{4x}]$

$y'' = A[16xe^{4x} + 8e^{4x}]$

$y'' = 16Axe^{4x} + 8Ae^{4x}$

Substitute into $y'' - 6y' + 8y = 8e^{4x}$

$16Axe^{4x} + 8Ae^{4x} - 6(4Axe^{4x} + Ae^{4x}) + 8(Axe^{4x})$
$= 8e^{4x}$

$8Ae^{4x} - 6Ae^{4x} = 8e^{4x}$

$2A\cancel{e^{4x}} = 8\cancel{e^{4x}}$

$A = 4$

$y = c_1 e^{4x} + c_2 e^{2x} + 4xe^{4x}$