NAME : WEST DA - ORIBIM

DEPT : MECH · ENG

1 $x = \cos t + t\sin t$

$y = \sin t - t\cos t$

a an expression for the radius of curvature (R) in terms of t

Ans

$x = \cos t + t\sin t$

$\frac{dx}{dt} = -\sin t + t\cos t + \sin t = t\cos t$

$y = \sin t - t\cos t$

$\frac{dy}{dt} = \cos t + t\sin t - \cos t = t\sin t$

$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dt}{dx}$

$= \frac{t\sin t}{t\cos t}$

$\frac{dy}{dx} = \frac{\sin t}{\cos t}$

$\frac{d^2y}{dx^2}$

$u = \sin t \qquad v = \cos t$

$du/dt = \cos t \qquad dv/dt = -\sin t$

$\frac{V\,du/dt - U\,dv/dx}{V^2}$

$\frac{\cos t\,(\cos t) - (\sin t)(-\sin t)}{(\cos t)^2} \times \frac{1}{t\cos t}$

$\frac{\cos^2 t + \sin^2 t}{\cos^2 t} \times \frac{1}{t\cos t}$

$\frac{1}{\cos^2 t} \times \frac{1}{t\cos t} = \frac{1}{t\cos^3 t}$

$\frac{d^2y}{dx^2} = \frac{1}{t\cos^3 t}$

$$R = \frac{\left[1 + \left(dy/dx\right)^2\right]^{3/2}}{d^2y/dx^2}$$

$$R = \frac{\left(1 + \left(\frac{\sin t}{\cos t}\right)^2\right)^{3/2}}{d^2y/dx^2}$$

$$R = \frac{\left(1 + \frac{\sin^2 t}{\cos^2 t}\right)^{3/2}}{\frac{1}{t\cos^3 t}}$$

$$R = \frac{\left(\frac{\cos^2 t + \sin^2 t}{\cos^2 t}\right)}{1/t\cos^3 t}$$

$$R = \frac{\left(1/\cos^2 t\right)^{3/2}}{1/t\cos^3 t}$$

$$R = \frac{1/\cos^3 t}{1/t\cos^3 t}$$

$$R = \frac{1}{\cancel{\cos^3 t}} \times t\cancel{\cos^3 t}$$

$$R = \underline{\underline{t}}$$

3 expressions for the co-ordinates (h, k) of the centre of curvature in terms of t

Ans

$$h = x_1 - R\sin t$$

$$k = y_1 + R\cos\theta$$

$\theta = t$

$h = x_1 - t \sin t$

$k = y_1 + t \cos t$

$=$

but $x = \cos t + t \sin t$

$y = \sin t - t \cos t$

$h = \cos t + t\sin t - t \sin t$

$h = \underline{\cos t}$

$k = \sin t - t\cos t + t \cos t$

$k = \sin t$

$(h, k) = (\cos t, \sin t)$