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15/ENG06/053
Mechanical Engineering
ENG 381

1) $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$

Solution

By taking the left hand side and equating to zero

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$$

$\therefore$ Auxilary equation ~~be~~ =

$$m^2 - m - 2 = 0$$

$$m^2 - 2m + m - 2 = 0$$

$$m(m-2) + 1(m-2) = 0$$

$$(m-\cancel{1}) + (m-2) = 0$$

$$(m+1)(m-2) = 0$$

$$m = -1 \quad \text{or} \quad m = 2$$

$$y = Ae^{-x} + Be^{2x}$$

Assume P I

$f(x) = 8$

$y = c$ ------ (I)

$\frac{dy}{dx} = 0$ ------ (II)

$\frac{d^2y}{dx} = 0$ ------ (III)

Substituting eqn (I), (II) & (III) into the original eqn.

$$0 - 0 - 2(c) = 8$$

$$-2c = 8$$

$$c = 8/2$$

$$c = 4$$

The general solution = Complementary function + assumed PI

$$y = Ae^{-x} + Be^{2x} - 4$$

2) $\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

Find C.F

$m^2 - 4 = 0$

$m^2 = 4$

$m = \pm\sqrt{4}$

$m = \pm 2$

$$y = Ae^{2x} + Be^{-2x}$$

$\therefore f(x) = 10e^{3x}$

$y = ce^{3x}$ ------ (i)

$\frac{dy}{dx} = 3ce^{3x}$ ------ (ii)

$\frac{d^2y}{dx^2} = 9ce^{3x}$ ------ (iii)

Substituting equation (i), (ii) & (iii) into the original eqn.

$9ce^{3x} - 4(ce^{3x}) = 10e^{3x}$

$9ce^{3x} - 4ce^{3x} = 10e^{3x}$

$5c\cancel{e^{3x}} = 10\cancel{e^{3x}}$

$c = \frac{10}{5}$

$c = 2$

3) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}$ Find C.f

$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$

Auxiliary eqn become =

$m^2 + 2m + 1 = 0$

$m = -1$ (twice) $y = e^{-x}[A + Bx]$

~~The thand~~ Assumed PI

$f(x) = e^{-2x}$

$y = Ce^{-2x}$ ---------- (i)

$\frac{dy}{dx} = -2Ce^{-2x}$ ---------- (ii)

$\frac{d^2y}{dx^2} = 4Ce^{-2x}$ ---------- (iii)

Substituting eqn. (i), (ii) & (iii) into the original equation

$4Ce^{-2x} + 2[-2Ce^{-2x}] + Ce^{-2x} = e^{-2x}$

$\cancel{4Ce^{-2x}} - \cancel{4Ce^{-2x}} + Ce^{-2x} - e^{-2x}$

$C\cancel{e^{-2x}} = \cancel{e^{-2x}}$

$C = 1$

$y = e^{-2x}$

The general solution = C.F + PI

$y = e^{-x}[A + Bx] + e^{-2x}$

4) $\frac{d^2y}{dx^2} + 25y = 5x^2 + x$

Solution

To find C.F

$\frac{d^2y}{dx^2} + 25y = 0$

The auxiliary equation becomes

$m^2 + 25 = 0$

$m^2 = -25$

$m = \pm\sqrt{-25}$

$m = \pm\sqrt{-1} \times \sqrt{25}$

$m = \pm j5$

Recall

$m = \alpha \pm jB$

$y = A\cos 5x + B\sin 5x$

Assumed PI

$fx = 5x^2 + x$, $y = Cx^2 + Dx + E$ ---- (i)

$$\frac{dy}{dx} = 2Cx + D \text{ ------ (ii)}$$

$$\frac{d^2y}{dx^2} = 2c \text{ ------------- (iii)}$$

Substituting eqn (i) & (iii) into the original equation

$$2C - 2[Cx^2 + Dx + E] = 5x^2 + x$$

$$2c - 2Cx^2 - 2Dx - 2E = 5x^2 + x$$

Comparing ceffficients

x^2: $2C = 5$

$C = 5/2$

x: $-2D = 1$

$D = -1/2$

K: $2C - 2E\ 0$

$2\left(\frac{5}{2}\right) - 2E = 0$

$5 = 2E$

$E = 5/2$

$\cancel{y = \frac{5x^2}{2} - x}$ $y = \frac{5x^2}{2} - \frac{x}{2} + \frac{5}{2}$

The general solution $= A\cos 5x + B\sin x + \frac{5x^2}{2} - \frac{x}{2} + \frac{5}{2}$ //