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16/ENG04/021
Elect/Elect Engineering
ENG 281 Assignment

1. Evaluate $\lim_{x \to \pi/2} \left[\frac{(x^2 - \pi/2)\sin(\cos x)}{x - \pi/2} \right]$

product rule $= V\frac{du}{dx} + U\frac{dv}{dx}$

$V = x^2 - \frac{\pi}{4}$, $U = \sin(\cos x)$

$P = \cos x$: $U = \sin P$

$\frac{dp}{dx} = -\sin x$ $\frac{du}{dp} = \cos P$

$\frac{du}{dx} = \frac{dp}{dx} \times \frac{du}{dp} = \frac{dp}{dx} \times \frac{du}{dx}$

$= -\sin x . \cos P$, $P = \cos x$

$\frac{du}{dx} = -\sin x - \cos(\cos x)$

$\frac{dv}{dx} = 2x$

$= (x^2 - \pi/2)(-\sin x \cdot \cos(\cos x)) + \sin(\cos x)(2x)$

$((\pi/2)^2 - \pi/4)(\sin(\pi/2)\cos(\cos \pi/2)) + \sin(\cos \pi/2)(2\pi/2)$

$= \pi/2 - \pi/2 \times -1$

$-\pi^2/4 + \pi/4$

$\lim_{x \to \pi/2} \left[\frac{(x^2 - \pi/2)(\sin \cos x)}{x - \pi/2} \right] = -\frac{\pi^2}{4} - \frac{\pi}{4}$

$\frac{\pi^2 - \pi}{4} = \frac{180^2 - 180}{4} = \cancel{8055}$

b. $\lim_{x \to \pi/2} \ln \left(\frac{\exp(3x^2+2x-1)}{x+1} \right)$

$\lim_{x \to \frac{\pi}{2}} (3x-1)$

$= 3\left(\frac{\pi}{2}\right)-1$

$= 3\left(\frac{\pi}{2}\right)-1$

$= 3\pi/2 - 1 = \frac{3\pi - 2}{2}$

$= \frac{66}{14} - 1 = 2\frac{6}{7}$

c. $\lim_{x \to 2\sqrt{3}} \cos \left(\sin^{-1} \frac{(x-2)}{(x-\sqrt{3})} \right)$

$\lim_{x \to 2+\sqrt{3}} \cos \sin^{-1} (x-2)$

$\lim_{x \to 2+\sqrt{3}} (\cos \sin^{-1} (x-\sqrt{3}))$

$= \cos \left[\sin^{-1} \frac{\sqrt{3}}{2} \right]$

$= \cos 60 = \frac{1}{2}$ or 0.5

d. $\lim_{x \to 4} \left(\frac{x^2 - 8x + 16}{x - 5x + 4} \right)$

$\lim_{x \to 4} \left(\frac{x^2 - 4x - 4x + 16}{x^2 - 4x - x + 4} \right)$

$\lim_{x \to 4} \left(\frac{(x^2 - 4x - 4x + 16}{x^2 - 4x - x + 4} \right)$

$\lim_{x \to 4} \left(\frac{x(x-4) - 4(x-4)}{x(x-4) - 1(x-4)} \right)$

$= \lim_{x \to 4} \left(\frac{x-4}{x-1} \right)$

$\frac{4-4}{4-1} = \frac{0}{3} = 0$

2. $\frac{2}{2 \times 3} + \frac{2}{3 \times 4} + \frac{2}{4 \times 5} + \frac{2}{5 \times 6} + \dots$

$u_n = \frac{2}{(n+1)(n+2)}$ $\quad u_{n+1} = \frac{2}{(n+2)(n+3)}$

$\lim_{n \to \infty} \frac{u_{n+1}}{u_n} = \frac{2}{(n+2)(n+3)} \times \frac{(n+1)(n+2)}{2}$

$\lim_{n \to \infty} \frac{u_{n+1}}{u_n} = \frac{n^2+3n+2}{n^2+5n+6}$

$\lim_{n \to \infty} \frac{1 + 3n/n^2 + 2/n^2}{n^2/n^2 + 5/n^2 + 6/n^2}$

$\frac{1+0+0}{1+0+0} = 1$

from $t_1 + 1$ $\lim u_n = \lim_{n \to \infty} \frac{2}{(n^2+3n+2)} = \frac{2/n^2}{n^2/n^2 + \frac{3}{n^2} + \frac{2}{n^2}}$

$0/1 = 0$

$\boxed{\text{The series is Convergent}}$