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 CIVIL ENGINEERING
 ENG 381

$$1 \quad \frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$$

Let $8 = 0$

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$$

$$m^2 - m - 2 = 0$$

$$m^2 + m - 2m - 2 = 0$$

$$m(m+1) - 2(m+1) = 0$$

$$(m-2)(m+1) = 0$$

$$m_1 = 2 \quad m_2 = -1$$

$$y = Ae^{2x} + Be^{-x}$$

$$P.I: y = c$$

$$\frac{dy}{dx} = 0$$

$$\frac{d^2y}{dx^2} = 0$$

$$0 - 0 - 2c = 8$$

$$-2c = 8$$

$$c = -4$$

$$P.I = -4$$

$$G.S = C.F + P.I$$

$$= Ae^{2x} + Be^{-x} - 4$$

$$2 \quad \frac{d^2y}{dx^2} - 4y = 10e^{3x}$$

Let $10e^{3x} = 0$

$$\frac{d^2y}{dx^2} - 4y = 0$$

$$m^2 - 4 = 0$$

$$m^2 = 4$$

$$m = \pm \sqrt{4}$$

$$m_1 = +2 \quad m_2 = -2 \quad m_3 = -2$$

$$y = Ae^{2x} + Be^{-2x}$$

$$P.I: y = Ce^{3x}$$

$$\frac{dy}{dx} = 3Ce^{3x}$$

$$\frac{d^2y}{dx^2} = 9Ce^{3x}$$

$$\frac{d^2y}{dx^2} - 4y = 10e^{3x}$$

$$9Ce^{3x} - 4(Ce^{3x}) = 10e^{3x}$$

$$9Ce^{3x} - 4Ce^{3x} = 10e^{3x}$$

Divide through by e^{3x}

$$9C - 4C = 10$$

$$5C = 10$$

$$C = 2$$

$$P.I = 2$$

$$G.S = C.F + P.I$$

$$G.S = Ae^{2x} + Be^{-2x} + 2e^{3x}$$

$$3 \quad \frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}$$

$$\text{Let } C = e^{-2x}$$

$$m^2 + 2m + 1 = 0$$

$$m^2 + m + m + 1 = 0$$

$$m(m+1) + 1(m+1) = 0$$

$$(m+1) = 0 \text{ twice}$$

$$m = -1 \text{ twice}$$

$$\text{C.F. : } y = e^{-x}(A + Bx)$$

$$\text{P.I. } y = Ce^{-2x}$$

$$\frac{dy}{dx} = -2Ce^{-2x}$$

$$dx$$

$$\frac{d^2y}{dx^2} = 4Ce^{-2x}$$

$$dx^2$$

$$4Ce^{-2x} + 2(-2Ce^{-2x}) + Ce^{-2x} = e^{-2x}$$

$$4Ce^{-2x} - 4Ce^{-2x} + Ce^{-2x} = e^{-2x}$$

divide through by e^{-2x}

$$4C - 4C + C = 1$$

$$C = 1$$

$$\text{G.S.} = \text{C.F.} + \text{P.I.}$$

$$\text{G.S.} = e^{-x}(A + Bx) + e^{-2x}$$

$$4 \quad \frac{d^2y}{dx^2} + 25y = 5x^2 + x$$

$$\text{let } 5x^2 + x = 0$$

$$m^2 + 25 = 0$$

$$m = \pm \sqrt{-25}$$

$$m = \pm j5$$

$$\text{C.F. : } y = A \cos 5x + B \sin 5x$$

$$\text{P.I. : } y = Cx^2 + Dx + E$$

$$\frac{dy}{dx} = 2Cx + D$$

$$dx$$

$$\frac{d^2y}{dx^2} = 2C$$

$$dx^2$$

$$* \frac{d^2 y}{dx^2} + 25y = 5x^2 + x$$

$$2C + 25(Cx^2 + Dx + E) = 5x^2 + x$$

$$2C + 25Cx^2 + 25Dx + 25E = 5x^2 + x$$

$$25C = 5 \dots (i)$$

$$25D = 1 \dots (ii)$$

$$2C + 25E = 0 \dots (iii)$$

from eqn (i)

$$C = \frac{5}{25} = \frac{1}{5}$$

from eqn (ii)

$$D = \frac{1}{25}$$

from eqn (iii)

$$2\left(\frac{1}{5}\right) + 25E = 0$$

$$\frac{2}{5} = -25E$$

$$\frac{0.4}{-25} = E$$

$$E = \frac{-2}{125}$$

$$P.I: y = Cx^2 + Dx + E$$

$$y = \frac{1}{5}x^2 + \frac{1}{25}x - \frac{2}{125}$$

$$G.S = C.F + P.I$$

$$= A \cos 5x + B \sin 5x + \frac{1}{5}x^2 + \frac{1}{25}x - \frac{2}{125}$$

$$5 \quad \frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$$

$$\text{Let } 4\sin x = 0$$

$$m^2 - 2m + 1 = 0$$

$$m^2 - m - m + 1 = 0$$

$$m(m-1) - 1(m-1) = 0$$

$$m-1 = 0 \text{ twice}$$

$$m = 1 \text{ twice}$$

$$\text{C.F.} \Rightarrow y = e^x(A+Bx)$$

$$F(x) = 4\sin x$$

$$\text{P.I.} \Rightarrow y = C\cos x + D\sin x$$

$$\frac{dy}{dx} = -C\sin x + D\cos x$$

$$\frac{d^2y}{dx^2} = -C\cos x - D\sin x$$

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$$

$$-C\cos x - D\sin x - 2(-C\sin x + D\cos x) + (C\cos x + D\sin x) = 4\sin x$$

$$-C\cos x - D\sin x + 2C\sin x - 2D\cos x + C\cos x + D\sin x = 4\sin x$$

$$+2C\sin x - 2D\cos x = 4\sin x$$

$$2C = 4 \quad \dots (1)$$

$$-2D = 0 \quad \dots (2)$$

$$D = 0$$

$$C = \frac{4}{2} = 2$$

$$\text{P.I.} \Rightarrow y = C\cos x + D\sin x$$

$$= 2\cos x + 0\sin x$$

$$= 2\cos x$$

$$\text{G.S.} = \text{C.F.} + \text{P.I.}$$

$$= e^x(A+Bx) + 2\cos x$$

$$6 \quad \frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$$

Given that at $x=0$, $y=1$ and $\frac{dy}{dx} = -2$

let $2e^{-2x} = 0$

$$m^2 + 4m + 5 = 0$$

$$a=1 \quad b=4 \quad c=5$$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$m = \frac{-4 \pm \sqrt{4^2 - 4 \times 1 \times 5}}{2 \times 1}$$

$$m = \frac{-4 \pm \sqrt{-4}}{2 \times 1}$$

$$m = \frac{-4 \pm j2}{2 \times 1}$$

$$m = -2 \pm j \quad \alpha = -2 \quad \beta = 1$$

$$C.F : y = e^{-2x} (A \cos x + B \sin x)$$

$$P.I : y = C e^{-2x}$$

$$\frac{dy}{dx} = -2C e^{-2x}$$

$$\frac{d^2y}{dx^2} = 4C e^{-2x}$$

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$$

$$4C e^{-2x} + 4(-2C e^{-2x}) + 5(C e^{-2x}) = 2e^{-2x}$$

$$4C e^{-2x} - 8C e^{-2x} + 5C e^{-2x} = 2e^{-2x}$$

divide through by e^{-2x}

$$4C - 8C + 5C = 2$$

$$C = 2$$

$$P.I \quad y = 2e^{-2x}$$

$$G.S = C.F + P.I$$

$$y = e^{-2x} (A \cos x + B \sin x) + 2e^{-2x}$$

When $x=0$ $y=1$

$$1 = e^{-2(0)} (A \cos(0) + B \sin(0)) + 2e^{-2(0)}$$

$$1 = 1(A + B(0)) + 2(1)$$

$$1 = A + 2$$

$$A = 1 - 2$$

$$A = -1$$

$$y = e^{-2x} (A \cos x + B \sin x) + 2e^{-2x}$$

$$u = e^{-2x}$$

$$\frac{du}{dx} = -2e^{-2x}$$

$$dx$$

$$v = A \cos x + B \sin x$$

$$\frac{dv}{dx} = -A \sin x + B \cos x$$

$$dx$$

$$\frac{dy}{dx} = e^{-2x} (-A \sin x + B \cos x) + (A \cos x + B \sin x) (-2e^{-2x}) - 4e^{-2x}$$

$$dx$$

When $x=0$ $\frac{dy}{dx} = -2$

$$dx$$

$$-2 = e^{-2(0)} (2(-1) \sin(0) + B \cos(0)) + (-1(\cos(0) + B \sin(0)) (-2) - 4$$

$$-2 = 1(0 + 2B) + (-1 + 0)(-2) - 4$$

$$-2 = 2B + 2 - 4$$

$$-2 = 2B - 2$$

$$2B = 0$$

$$B = 0$$

$$A = -1 \quad B = 0$$

$$y = e^{-2x} (A \cos x + B \sin x) + 2e^{-2x}$$

$$y = e^{-2x} (-1 \cos x + 0 \sin x) + 2e^{-2x}$$

$$y = e^{-2x} (-\cos x) + 2e^{-2x}$$

$$y = e^{-2x} (2 - \cos x)$$

$$7 \quad 3 \frac{d^2y}{dx^2} - 2 \frac{dy}{dx} - y = 2x - 3$$

$$\text{let } 2x - 3 = 0$$

$$3m^2 - 2m - 1 = 0$$

$$3m^2 - 3m + m - 1 = 0$$

$$3m(m-1) + 1(m-1) = 0$$

$$(m-1)(3m+1) = 0$$

$$m_1 = 1 \quad 3m_2 = -1$$

$$m = -\frac{1}{3}$$

$$\text{CF: } y = Ae^x + Be^{-x/3}$$

$$P(x) = 2x - 3$$

$$y = Cx^2 + Dx + E$$

$$\frac{dy}{dx} = 2Cx + D$$

$$dx$$

$$\frac{d^2y}{dx^2} = 2C$$

$$dx^2$$

$$3(2C) - 2(2Cx + D) - (Cx^2 + Dx + E) = 2x - 3$$

$$6C - 4Cx - 2D - Cx^2 - Dx - E = 2x - 3$$

$$-Cx^2 = 0$$

$$C = 0$$

$$-4C - D = 2$$

$$-4(0) - D = 2$$

$$-D = 2$$

$$D = -2$$

$$6C - E - 2D = -3$$

$$6(0) - E - 2(-2) = -3$$

$$-E + 4 = -3$$

$$-E = -7$$

$$E = 7$$

$$y = 0x^2 - 2x + 7$$

$$y = -2x + 7$$

$$y = Ae^x + Be^{-x/3} - 2x + 7$$

$$2 \quad \frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$$

$$\text{Let } 8e^{4x} = 0$$

$$m^2 - 6m + 8 = 0$$

$$m^2 - 4m - 2m + 8 =$$

$$m(m-4) - 2(m-4) =$$

$$(m-4)(m-2) = 0$$

$$m_2 = 4 \quad m_1 = 2$$

$$C.F = Ae^{2x} + Be^{4x}$$

$$y = Ce^{4x}$$

$$\frac{dy}{dx} = 4Ce^{4x}$$

$$dx$$

$$\frac{d^2y}{dx^2} = 16Ce^{4x}$$

$$dx^2$$

$$16Ce^{4x} - 6(4Ce^{4x}) + 8(Ce^{4x}) = 8e^{4x}$$

$$16Ce^{4x} - 24Ce^{4x} + 8Ce^{4x} = 8e^{4x}$$

C is undefined

$$\text{Therefore } y = Cxe^{4x}$$

$$u = Cx \quad v = e^{4x}$$

$$\frac{du}{dx} = C \quad \frac{dv}{dx} = 4e^{4x}$$

$$dx \quad dx$$

$$\frac{dy}{dx} = Cx(4e^{4x}) + e^{4x}(C)$$

$$dx$$

$$= 4Cx e^{4x} + C e^{4x}$$

$$\frac{d^2y}{dx^2} = 16Cx e^{4x} + 4C e^{4x} + 4C e^{4x}$$

$$dx^2 = 16Cx e^{4x} + 8C e^{4x}$$

$$y = 16Cx e^{4x} + 8C e^{4x} - 6(4Cx e^{4x} + C e^{4x}) + 8Cx e^{4x} = 8e^{4x}$$

$$y = 16Cx e^{4x} + 8C e^{4x} - 24Cx e^{4x} - 6C e^{4x} + 8Cx e^{4x} = 8e^{4x}$$

$$2C e^{4x} = 8e^{4x}$$

$$C = 4$$

$$y = 4xe^{4x}$$

$$C.F = Ae^{2x} + Be^{4x} + 4xe^{4x}$$