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16/ENG06/058
Mechanical Engineering
Eng 281

1.

a. $\lim_{x \to \frac{\pi}{2}} \left[\frac{\left[x^2 - \frac{\pi}{4}\right] \sin(\cos x)}{x - \pi/2} \right]$

$\lim_{x \to \pi/2} \left[\frac{\left((\pi/2)^2 - \frac{\pi}{4}\right) \sin\left(\cos \frac{\pi}{2}\right)}{\pi/2 - \frac{\pi}{2}} \right] = \text{undefined}$

$y = \sin(u) \qquad u = \cos(x)$

$\frac{dy}{du} = \cos(u) \qquad \frac{du}{dx} = -\sin(x)$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \cos(u) \times -\sin(x) = \cos(\cos(x)) \times -\sin(x)$

$F(x) = x^2 \cos(\cos x) \sin x + 2x \sin(\cos x) + \frac{\pi}{4} \cos(\cos x) \sin x$

$\lim_{x \to \pi/2} F'(x) = -\frac{\pi^2}{4} \cos\left(\cos \frac{\pi}{2}\right) \sin\left[\frac{\pi}{2}\right] + 2\left[\frac{\pi}{2}\right] \sin\left[\cos\left(\frac{\pi}{2}\right)\right] + \frac{\pi}{4} \cos \left[\cos \frac{\pi}{2}\right] \sin\left[\frac{\pi}{2}\right]$

$= -\frac{\pi^2}{4} \cos(0) \times 1 + \sin(0) \times 180 + \frac{\pi}{4} \cos(0) \times 0$

$= -\frac{\pi^2}{4} \times 1 + \frac{\pi}{4} \times 1$

$= -\frac{\pi^2}{4} + \frac{\pi}{4} = \frac{-\pi^2 + \pi}{4}$

$\lim_{x \to \frac{\pi}{2}} \left[\frac{\left[x^2 - \pi/4\right] \sin(\cos x)}{x - \pi/2} \right] = \frac{-\pi^2 + \pi}{4}$

1b $\lim_{x \to \pi/2} \ln\left[\exp\left(\frac{3x^2+2x-1}{x+1}\right)\right]$

$$\frac{3(\pi/2)^2 + 2(\pi/2) - 1}{(\pi/2)+1} = 3.71$$

$$= \ln\left[\exp(3.71)\right]$$

$$= \ln[40.85]$$

$$= 3.71$$

1c $\lim_{x \to 2+\sqrt{3}} \cos\left[\sin^{-1}\frac{(x-2)}{(x-\sqrt{3})}\right]$

$$\lim_{x \to 2+\sqrt{3}} \cos\left[\sin^{-1}\frac{(2+\sqrt{3}-2)}{(2+\sqrt{3}-\sqrt{3})}\right]$$

$$\lim_{x \to 2+\sqrt{3}} \cos\left[\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right]$$

$$\cos(60)$$

$$\lim_{x \to 2+\sqrt{3}} \cos\left[\sin^{-1}\frac{(x-2)}{(x-\sqrt{3})}\right] = \frac{1}{2}$$

1d $\lim_{x \to 4} \left[\frac{x^2 - 8x + 16}{x^2 - 5x + 4}\right]$

$$\lim_{x \to 4} \left[\frac{4^2 - 8(4) + 16}{4^2 - 5(4) + 4}\right] = \text{undefined}$$

$$\lim \left[\frac{2x - 8}{2x - 5}\right]$$

$$\lim_{x \to 4} \left[\frac{2(4) - 8}{2(4) - 5}\right] = 0$$

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2a $\frac{2}{2\times3} + \frac{2}{3\times4} + \frac{2}{4\times5} + \frac{2}{5\times6}$

taking standard series

$\frac{2}{2^p} + \frac{2}{3^p} + \frac{2}{4^p} + \frac{2}{5^p}$

when $p = 2$

$\frac{2}{2\times3} < \frac{2}{2^2}$, $\frac{2}{3\times4} < \frac{2}{3^2}$

it converges

b $\frac{2}{1^2} + \frac{2}{2^2} + \frac{2}{3^2} + \frac{2}{4^2} + \dots + \frac{2}{n^p}$

Recall if $P>1$ it converges

if $P \le 1$ it diverges

$\frac{2}{n^2}$ $P = 2$ The series converges

2c $u_n = \frac{1+2n^2}{1+n^2}$

$u_{n+1} = \frac{1+2(n+1)^2}{1+(n+1)^2} = \frac{2n^2+4n+3}{n^2+2n+2}$

$\frac{u_{n+1}}{u_n} = \frac{2n^2+4n+3}{n^2+2n+2} \times \frac{1+n^2}{1+2n^2}$

$= \frac{(1+n^2)(2n^2+4n+3)}{(1+2n^2)(n^2+2n+2)}$

$= \frac{2n^4+4n^3+5n^2+4n+3}{2n^4+4n^3+5n^2+2n+2}$

$= \frac{4n+3}{2n+3} = \frac{4+3/n}{2+3/n} = \frac{4}{2}$

$\frac{4}{2} = 2 > 1$: it converges

3. $\frac{x}{27} + \frac{x^2}{125} + \dots + \frac{x^n}{(2n+1)^3}$

$|u_n| = \frac{x^n}{(2n+1)^3}$

$|u_{n+1}| = \frac{x^{n+1}}{(2n+3)^3}$

$\frac{u_{n+1}}{u_n} = \frac{x^{n+1}}{(2n+3)^3} \times \frac{(2n+1)^3}{x^n}$

$= \frac{x^{n+1}}{2n^3 + 18n^2 + 54n + 9} \times \frac{2n^3 + 6n^2 + 6n + 1}{x^n}$

$= \frac{x(6n^2 + 6n + 1)}{18n^2 + 54n + 9}$

$= x(6 + 6/n + 1/n^2)$

$\frac{u_{n+1}}{u_n} = \frac{6x}{18}$

$\frac{6x}{18} < 1$

$6x < 18$

$|x| < \frac{18}{6}$

$|x| < 3$

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4. $\lim_{x \to 0} \left[\frac{\sin x - \cos x}{x^3} \right]$

$\lim_{x \to 0} \left[\frac{\sin(0) - \cos(0)}{(0)^3} \right] = \text{Undefined}$

from L'Hopital Law

$\lim_{x \to 0} \left[\frac{\cos x + \sin x}{3x^2} \right]$

$\lim_{x \to 0} \left[\frac{\cos(0) + \sin(0)}{3(0)^2} \right]$

$\lim_{x \to 0} \left[\frac{-\sin x + \cos(x)}{6x} \right]$

$\lim_{x \to 0} \left[\frac{-\sin(0) + \cos(0)}{6(0)} \right] = \text{Undefined}$

$\lim_{x \to 0} \left[\frac{-\cos(x) - \sin x}{6} \right]$

$\lim_{x \to 0} \left[\frac{\cos(0) - \sin(0)}{6} \right] = -\frac{1}{6}$