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15/ENG03/022
CIVIL ENGINEERING
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i) $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 5$

$m^2 - m - 2 = 0$

$(m^2 - 2m) + (m - 2) = 0$

$m(m-2) + 1(m-2) = 0$

$(m+1)(m-2) = 0$

$m = -1$ and 2

$y = Ae^{2x} + Be^{-x}$

$\frac{dy}{dx} = C$

$\frac{dy}{dx} = 0$

$\frac{d^2y}{dx^2} = 0$

$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 5$

$0 - 0 - 2C = 5$

$-2C = 5$

$C = -4$

$PI = y = -4$

$\therefore y = Ae^{2x} + Be^{-x} - 4$

ii) $\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

$\frac{d^2y}{dx^2} - 4y = 0$

$m^2 - 4 = 0$

$m^2 = \pm 4$

$m = \pm\sqrt{4}$

$m = \pm 2$

$C.F = A\cosh 2x + B\sinh 3x$

$P.I = y = Ce^{3x}$

$\frac{dy}{dx} = 3Ce^{3x}$

$\frac{d^2y}{dx^2} = 9Ce^{3x}$

15/ENG03/022

$\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

$9Ce^{3x} - 4Ce^{3x} = 10e^{3x}$

$5Ce^{3x} = 10e^{3x}$

$C = 2$

$y = 2e^{3x}$

C.S = A.S + P.S

C.S $= Ae^{2x} + Be^{-2x} + 2e^{3x}$

3) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-3x}$

$m^2 + 2m + 1 = 0$

$(m^2 + m)(m + 1) = 0$

$m(m+1) + 1(m+1) = 0$

$(m+1)(m+1) = 0$

$m = -1$ (two roots).

$y = e^{-3x}(Ax + B)$

P.I $= y = Ce^{-3x}$

$\frac{dy}{dx} = -3Ce^{-3x}$

$\frac{d^2y}{dx^2} = 9Ce^{-3x}$

$= 9Ce^{-3x} + 2(-3Ce^{-3x}) + Ce^{-3x} = e^{-3x}$

$9Ce^{-3x} - 6Ce^{-3x} + Ce^{-3x} = e^{-3x}$

$Ce^{-3x} = e^{-3x}$

$C = 1$

$y = e^{-3x}$

G.S $= e^{-3x}(Ax + B) + e^{-3x}$

4) $\frac{d^2y}{dx^2} + 25y = 5x^2 + 7$

$\frac{d^2y}{dx^2} + 25y = 0$

$m^2 + 25 = 0$

15/ENG03/022.

$m^2 = \pm\sqrt{25}$

$m = \pm 5$

C.F = $A\cos 5x + B\sin 5x$

$y = Cx^2 + Dx + E$

$\frac{dy}{dx} = 2Cx + D$

$\frac{d^2y}{d^2x} = 2C$

$2C + 25(Cx^2 + Dx + E) = 5x^2 + x$

$2C + 25Cx^2 + 25Dx + 25E = 5x^2 + x$

$2C + 25E + 25Cx^2 + 25Dx = 5x^2 + x$

$25C = 5$

$C = 5/25 = 1/5$

$25D = 1$

$D = 1/25$

$2C + 25E = 0$

$2(1/5) + 25E = 0$

$2/5 = -25E$

$E = -2/125$

$y = 1/5x^2 + 1/25x - 2/125$

G.S = $A\cos 5x + B\sin 5x + 1/5(x^2 + 1/5x - \frac{2}{25})$

5) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$

$m^2 - 2m + 1 = 0$

$m^2 - m - m + 1 = 0$

$m(m-1) - 1(m-1) = 0$

$m = 1$ twice

$y = e^x(A + Bx)$

$y = C\cos x + D\sin x$

$\frac{dy}{dx} = -C\sin x + D\cos x$

$D = 0$

$y = Ce^{2x} + Dxe^{2x}$

$y' = 2Ce^{2x} + De^{2x} = 2Ce^{2x}$

$C.S = e^{2x}(A + Bx) + 2e^{-2x}$

Q) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$ given that $x = 0, y = 1$ and $\frac{dy}{dx} = -2$

$m^2 + 4m + 5 = 0$

$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-4 \pm \sqrt{4^2 - 4 \times 1 \times 5}}{2 \times 1}$

$= \frac{-4 \pm \sqrt{16 - 20}}{2}$ $= \frac{-4 \pm \sqrt{-4}}{2}$ $= \frac{-4 \pm j\sqrt{4}}{2}$

$m = \frac{-4}{2} \pm \frac{j\sqrt{4}}{2} = -2 \pm j$

$\alpha = -2, \beta = 1$

$C.F = y = e^{-2x}(A\cos x + B\sin x)$

$y = Ce^{-2x}$

$\frac{dy}{dx} = -2Ce^{-2x}$

15/ENG03/022
Mathematical Methods.

$\frac{d^2y}{dx^2} = 4Ce^{-2x}$

$= 4Ce^{-2x} - 2(e^{-2x}(4)) + 5Ce^{-2x} = 2e^{-2x}$

$4Ce^{-2x} - 8Ce^{-2x} + 5Ce^{-2x} = 2e^{-2x}$

$Ce^{-2x} = 2e^{-2x}$

$C = 2$

$y = 2e^{-2x}$

G.S $= e^{-2x}(A\cos 2x + B\sin 2x) + 2e^{-2x}$

given that $x = 0$, $y = 1$ and $\frac{dy}{dx} = -2$

$y = Ae^{-2x}\cos 2x + Be^{-2x}\sin 2x + 2e^{-2x}$

$1 = Ae^{-2(0)}\cos 0 + Be^{-2(0)}\sin 0 + 2e^{-2(0)}$

$1 = A + 2$

$A = 1 - 2 = -1$

$\frac{dy}{dx} = -2Ae^{-2x}\cos 2x - \sin 2x Ae^{-2x} - 2Be^{-2x}\sin 2x + \cos 2x Be^{-2x}$
$- 4e^{-2x}$

$-2 = -2Ae^{-2(0)} + Be^{-2(0)} - 4e^{-2(0)}$

$-2 = -2A + B - 4$

$-2 = -2(-1) + B - 4$

$-2 = +2 + B - 4 \quad \therefore -2 + 2 = B \therefore B = 0$

$\therefore y = e^{-2x}(-\cos 2x + 0\sin 2x) + 2e^{-2x}$

$y = e^{-2x}\cos 2x + 2e^{-2x} = e^{-2x}(-\cos 2x + 2)$

7) $3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 2x - 3$

$3m^2 - 2m - 1 = 0$

$3m^2 - 3m + 1m - 1 = 0$

$3m(m-1) + 1(m-1) = 0$

$(3m+1)(m-1) = 0$

$m = -1/3$ & 1

$y = Ae^{-1/3x} + Be^{x}$

$\frac{dx}{}$

$\frac{d^2y}{dx^2} = 0$

$\frac{3\partial^2 y}{\partial x^2} - \frac{2\partial y}{\partial x} - y = 2x - 3$

$3(0) - 2c - ((x+D) = 2x - 3$

$-2c - cx - D = 2x - 3$

Compare the two sides.

$-cx = 2x$

$c = -2$

$-2c - D = -3$

$-2(-2) - D = -3$

$4 - D = -3$

$-D = -3 - 4$

$-D = -7$

$D = 7$

$y = -2x + 7$

G.S $= Ae^{2x/3} + Be^{2x} - 2x + 7$

(2) $\frac{\partial^2 y}{\partial x^2} - 6\frac{\partial y}{\partial x} + 8y = 8e^{4x}$

$m^2 - 6m + 8 = 0$

$(m^2 - 4m)(-2m + 8) = 0$

$m(m-4) - 2(m-4) = 0$

$(m-2)(m-4) = 0$

$m = 2$ and 4

$y = Ae^{2x} + Be^{4x}$

16/ENR03/022

$\frac{d^2y}{dx^2} = 16Ce^{4x}$

$16Ce^{4x} - 6(4Ce^{4x}) + 8(Ce^{4x}) = 8e^{4x}$

$16Ce^{4x} - 24Ce^{4x} + 8Ce^{4x} = 8e^{4x}$

C is undefined.

assume the particular therefore $y = Cxe^{4x}$

use product rule to differentiate.

$u = Cx \quad v = e^{4x}$

$\frac{du}{dx} = C \quad \frac{dv}{dx} = 4e^{4x}$

$\frac{dy}{dx} = Cx(4e^{4x}) + e^{4x}(C)$

$= 4Cxe^{4x} + Ce^{4x}$

$\frac{d^2y}{dx^2} = 16Cxe^{4x} + 4Ce^{4x} + 4Ce^{4x}$

$= 16Cxe^{4x} + 8Ce^{4x}$

$= 16Cxe^{4x} + 8Ce^{4x} - 6(4Cxe^{4x} + Ce^{4x}) + 8(Cxe^{4x}) = 8e^{4x}$

$= 16Cxe^{4x} + 8Ce^{4x} - 24Cxe^{4x} - 6Ce^{4x} + 8Cxe^{4x} = 8e^{4x}$

$2Ce^{4x} = 8e^{4x}$

$C = 4$

$y = 4xe^{4x}$

G.S $= Ae^{2x} + Be^{4x} + 4xe^{4x}$