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15/ENG01/026
Chemical Engineering

ENG 381

1 $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$

$\Rightarrow y'' - y' - 2y = 8$

let y

∴ auxiliary equation $m^2 - m - 2 = 0$

$(m-2)(m+1) = 0$

∴ $m = 2$ or -1

∴ $CF = Ae^{2x} + Be^{-x}$

$PI = y_p = C$

$y_p = 0$

$y'_p = 0$

$y''_p = 0$

∴ $0 - 0 - 2C = 8$

$C = -4$

∴ $y_p = -4$

∴ G.S, $y = Ae^{2x} + Be^{-x} - 4$

2 $\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

auxiliary equation =

$m^2 - 4 = 0$

∴ $m = \sqrt{4} = \sqrt{-1} \times \sqrt{4}$

$= \pm 2i$

$C.F = y_c = A\cos 2x + B\sin 2x$

$PI, y_p = Ce^{3x}$

$y'_p = 3Ce^{3x}$

$y''_p = 9Ce^{3x}$

$9Ce^{3x} - 4Ce^{3x} = 10e^{3x}$

$5Ce^{3x} = 10e^{3x}$

$C = 2$

∴ G.S $= A\cos 2x + B\sin 2x + 2e^{3x}$

3 $3\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}$

auxiliary equation $m^2 + 2m + 1 = 0$

$(m+1)(m+1) = 0$

$m = -1$

∴ Complementary function $y_c = Ae^{-x} + Bxe^{-x}$

$PI, y_p = Ce^{-2x}$

∴ $y'_p = -2Ce^{-2x}$

∴ $y''_p = 4Ce^{-2x}$

$4Ce^{-2x} + 2(-2Ce^{-2x}) + Ce^{-2x} = e^{-2x}$

dividing through by e^{-2x}

$C = 1$

∴ $Ae^{-x} + Bxe^{-x} + e^{-2x}$

$\frac{d^2y}{dx^2} + 25y = 5x^2 + x$

auxiliary equation: $m^2 + 25$

∴ $m = \sqrt{-25} = \sqrt{-1} \times \sqrt{25} = \pm 5i$

∴ $C.F, y_c = A\cos 5x + B\sin 5x$

$PI, y_p = Cx^2 + Dx + E$

$y'_p = 2Cx + D$

$y''_p = 2C$

Substituting values

$2C + 25(Cx^2 + Dx + E) = 5x^2 + x$

$2C + 25x^2 + 25Cx + E = 5x^2$

$\therefore 2C + E = 0$

$25Cx - x = 0$

$25Cx^2 = 5x^2$

$5C = 1$

$C = \frac{1}{5}$

$\therefore \frac{2}{5} + E = 0$

$E = -\frac{2}{5}$

$25Dx = x$

$25D = 1 \Rightarrow D = \frac{1}{25}$

$\therefore C.S = A\cos 5x + B\sin 5x + \frac{1}{5}x^2 + \frac{1}{25}x - \frac{2}{5}$

5 $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$

auxiliary equation: $m^2 - 2m + 1 = 0$

$\therefore (m-1)(m-1) = 0$

$\therefore m = 1$

$\therefore C.F\ y_c = Ae^x + Bxe^x$

$P.I\ y_p = C\sin x + D\cos x$

$y_p' = C\cos x - D\sin x$

$y_p'' = -C\sin x - D\cos x$

$\therefore -C\sin x - D\cos x - 2[C\cos x - D\sin x] + C\sin x + D\cos x = 4\sin x$

$2D\sin x - 2C\cos x = 4\sin x$

$2D\sin x = 4\sin x$

$-2C\cos x = 0$

$D = 2$

$C = 0$

$\therefore y_p = 2\sin x$

$\therefore G.S = Ae^x + Bxe^x + 2\sin x$

7 $\frac{3d^2y}{dx^2} - 2\frac{dy}{dx} + y = 2x - 3$

auxiliary equation: $3m^2 - 2m + 1 = 0$

$\therefore (3m-1)(m-1) = 0$

$\therefore m = \frac{1}{3}$ or 1

$C.F,\ y_c = Ae^{\frac{1}{3}x} + Be^x$

$P.I\ y_p = Cx + D$

$y_p' = C$

$y_p'' = 0$

$0 - 2x - 3 = -2Cx - C + D$

$2x - 3 = -3Cx - D$

$\therefore 2x = -3Cx$

$D = 3$

$C = -\frac{2}{3}$

$\therefore C.S = Ae^{\frac{1}{3}x} + Be^x + \frac{2}{3}x - \frac{3}{3}$

8 $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$

auxiliary equation: $m^2 - 6m + 8 = 0$

$\therefore (m-4)(m-2) = 0$

$m = 4$ or 2

$\therefore C.F.\ y_c = Ae^{4x} + Be^{2x}$

$P.I,\ y_p = Ce^x$

$y_p' = 4Ce^x$

$y_p'' = 16Ce^x$

$\therefore 16Ce^x - 24Ce^x + 8Ce^x = 8e^x$

$20Ce^x = 8e^x$

$C = \frac{2}{5}$

$\therefore C.S = Ae^{4x} + Be^{2x} + \frac{2}{5}e^x$